LOW FREQUENCY WHISTLERS GENERATED BY INFRASONIC WAVES IN THE IONOSPHERIC E-REGION DURING DISTURBANCES OF DIFFERENT NATURE

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Abstract. It is shown that infrasonic waves generated in the atmosphere during disturbances of different natures can generate or enhance low frequency whistlers in the ionospheric E-region. From the dispersion equation it is derived a relation between frequencies of infrasound and those of a whistler; using the papers dealing with experimental data on an interval of frequencies of infrasound generated in the atmosphere during disturbances of different natures, the frequency interval, in which generation of whistlers is possible, is calculated.

Key words: infrasound, whistlers, ionospheric E-region

1. Introduction

As is known, infrasonic waves in the range of $0.01 \, \text{Hz} < f_i < 20 \, \text{Hz}$ are generated during strong thunderstorms, explosions, earthquakes and other disturbances (see, for example, Grigor'ev and Dolushayev (1981); Alperovich et al. (1983)). Such waves reach the ionospheric E-region heights ($z \sim 100$-$160 \, \text{km}$) rather freely, resulting in appearance of additional currents and in disturbing electric and magnetic fields, i.e. in generating or enhancing different waves. It is also known that whistlers (whistler modes) are electromagnetic waves propagating in weakly-ionized plasma for which the following conditions are fulfilled:

$$v_i \ll \omega_i, \omega_q = \omega_{i} - \omega_{e}$$

($\omega_i$, $\omega_q$ are the electron and ion gyrofrequencies, $v_i$, $v_q$ are the electron and ion collision-frequencies). For such waves, the electrons taking part in infrasonic wave motions are related to magnetic lines, drifting in crossed fields; as to the ions, they, due to their collisions with neutrals, are not subjected to magnetic field effects, being practically at rest or moving under the infrasonic wave control. The conditions are valid at the ionospheric E-region heights. The presence of such waves in the epicentral zone and at some distance from a disturbance source was found in the experiment (see, for example, Mikhaylov et al. (1997)). The question of the low frequency whistlers propagating in that height range were discussed rather fully in Mazur (1988). The problem of such waves generated by acoustic waves in the E-region was solved in Surkov (1989).

In this paper, we demonstrate a possibility of generating low frequency whistlers by infrasound.

2. Main part

An initial equation system consists of Maxwell equations (2), dynamic ones (conserving momentum of the particles) (3-4) and that of continuity (5):
\[
\frac{\partial B}{\partial t} = (4\pi / c) J, \quad \frac{\partial E}{\partial t} = -(1 / c) \frac{\partial B}{\partial t}, \quad J = \sigma E,
\]
(2)

\[
\frac{\partial \mathbf{V}_x}{\partial t} + (\mathbf{V}_x \cdot \nabla) \mathbf{V}_x = -\frac{q_e}{m_e} \left( E + \frac{1}{c} [\mathbf{V}_x, \mathbf{B}] \right) - v_n \mathbf{V}_x \cdot \mathbf{V}_x - v_i \mathbf{V}_i \cdot \mathbf{V}_x - (\mathbf{V}_x - \mathbf{V}_z).
\]
(3)

\[
\frac{\partial \mathbf{V}_i}{\partial t} + (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i = -\frac{q_i}{m_i} \left( E + \frac{1}{c} [\mathbf{V}_i, \mathbf{B}] \right) - v_n \mathbf{V}_n \cdot \mathbf{V}_i - v_i \mathbf{V}_i \cdot \mathbf{V}_i - (\mathbf{V}_i - \mathbf{V}_x).
\]
(4)

\[
\frac{\partial N_n}{\partial t} + d \mathbf{V}_n \cdot \nabla N_n = 0.
\]
(5)

Here \( q_e, m_e \) are the charge and mass of electrons, \( m_i \) is the ion mass, \( \alpha = e, i \), the \( n \)-index corresponds to neutrals, \( \mathbf{V}_n, \mathbf{V}_a \) being their hydrodynamic velocities. Consider a two-component plasma. When solving equations (2)-(5), we use a linear approximation for the equations in the Gauss coordinate system which is chosen as follows: the wave vector \( \mathbf{k} \) coincides with the axis \( z \), the magnetic induction vector \( \mathbf{B} \) being in the \( xy \)-plane, the angle \( \mathbf{k} \mathbf{B} = \theta \). \( \mathbf{V}_n \) changes and intensities of the electric field \( \mathbf{E} \) are of wave character, i.e. \( \mathbf{V}_n, \mathbf{E} = \exp \left( i(\omega t - k r) \right) \). Suppose the ion mass being rather large, which allows us to neglect magnetic field effects on ions. Constrained oscillations of ionospheric plasma as the result of affecting it by an infrasonic wave are considered as a quasi-stationary background if compared with natural plasma-oscillations. Taking into account the above-stated conditions, we shall write down equations (3-4) in linear approximation as:

\[
\frac{\partial \mathbf{V}_n}{\partial t} = \frac{q_e}{m_e} \left( E + \frac{1}{c} [\mathbf{V}_n, \mathbf{B}] \right) - \frac{\nabla p_n}{m_n N_n} v_n \mathbf{V}_n \mathbf{V}_x - \mathbf{V}_n \mathbf{V}_x + v_n (\mathbf{V}_n - \mathbf{V}_n).
\]
(6)

Here \( p_n \) is the pressure change for particles of the \( \alpha \)-species, caused by an infrasonic wave. A current expression will be

\[
\mathbf{J} = \sum_{\alpha} q_{\alpha} N_{\alpha} \mathbf{V}_{\alpha}.
\]
(7)

where \( N_{\alpha} \) is the density of particles. Consider wave oscillations in plasma having a spatial irregularity \( L \sim \Lambda_1, \) \( (\Lambda_1 \text{ is the infrasonic wavelength}) \), whose propagation velocities are far larger than those of infrasonic waves, \( \mathbf{V}_n \). Then \( v_n \mathbf{V}_n \) and \( \nabla p_n / m_n N_n \) in (6) may be neglected as they do not change much over a period of natural plasma-oscillations. From (6), an expression for \( \mathbf{V}_n \) is taken as:

\[
\mathbf{V}_n = \frac{1}{j v_{an} - m} \left( a_n E + \frac{1}{c} a_n [\mathbf{V}_n, \mathbf{B}_n] + v_n \mathbf{V}_n - \frac{\nabla p_n}{m_n N_n} \right).
\]
(8)

here \( a_{\alpha} = q_{\alpha} / m_{\alpha} \) are the charge and the mass of the \( e, i \)-species. Further, after multiplying (8) by \( \mathbf{B} \) in scalar and vectorial ways, we find an expression for \( [\mathbf{V}_n, \mathbf{B}] \) and substitute it into (8) (we leave out these obvious cumbersome transformations). We shall sub-
stitute the expression derived for $V_\alpha$ in such a way into (7) and write down the resulting expression for current in a matrix form in the coordinate system chosen

$$j = \mathbf{A} (E + \alpha_m A_\alpha),$$

$$\mathbf{A} = \begin{pmatrix} A & -iF \cos \theta & iF \sin \theta \\ iF \cos \theta & A - C \sin^2 \theta & -0.5C \sin 2\theta \\ -iF \sin \theta & -0.5C \sin 2\theta & D - C \sin^2 \theta \end{pmatrix},$$

here $A_\alpha = A_\alpha(\omega, \omega_\nu, \omega_\pi, \omega_\nu)$ and the following designations are introduced:

$$A = \sum \frac{\omega^2_\nu}{\omega} (M + K), \quad C = A - (1 + D), \quad D = \sum \frac{\omega^2_\nu}{\omega} (K - M),$$

$$M = 1 / ((\omega - i\nu_m) + \omega), \quad K = 1 / ((\omega - i\nu_m) - \omega), \quad F = \sum \frac{\omega^2_\nu}{2\omega} (K - M),$$

$$q^2 = q^2 + \omega^2_\nu = 4\pi q^2 N_\nu / m \omega_\nu \omega_\nu = q_\alpha^2 / c m_\alpha (\omega_\nu, \omega_\nu)$$

Methods of deriving the dispersion equation are similar to those in Ginzburg and Rukhadze (1970), and therefore we omit cumbersome intermediate calculations. Solving (2) similarly to that in Ginzburg and Rukhadze (1970), leads to the following expression:

$$k^2 E + (kE)k - (\omega^2 / c^2) E + (i4\pi \omega / c^2) j = 0,$$

A substitution of current expression (9) into (11) makes it inhomogeneous. A general solution of (11) in the given case comes to solving a homogeneous equation as a particular solution of an inhomogeneous equation may be neglected owing to the suggestions considered above. It allows us to find the dispersion equation in a general form similar to that given in Ginzburg and Rukhadze (1970) as $\tau^2 = (\beta_1 \pm C_1) / A_1$ (where the values of $A_1, B_1, C_1$ are the cumbersome functions $A_1, B_1, C_1 = f(\omega_\nu, \omega_\nu, \omega_\nu, \omega_\nu, \omega_\nu, \nu_m, \nu_m, \nu_m)$ in the designations given earlier; we omit them in order not to burden the text with unnecessary detail), from which for waves with $\omega_\nu << \omega << \omega_\nu$ (low frequency whistlers) we shall determine the dispersion equation (the two known cases are not considered as they do not come into our task) as

$$n^2_1(\omega) = \omega^2 / \omega_\nu \omega_\nu \cos \theta.$$

A spectrum of such waves is determined from (9) as follows:

$$\omega_1(k) = |\omega_\nu \cos \theta| k^2 c^2 / \omega_\nu^2.$$

Waves described by dispersion relation (13) are purely electron ones and can propagate in ionospheric plasma in a narrow cone of angles having its axis along the magnetic field (Mazur (1988); Ginzburg and Rukhadze (1970)). The solution of (13) allows to find the relation be-
between \( f_i \) and the frequencies of the low frequency whistlers generated in this height range, \( f_j \), and make calculations:

\[
f_j = \frac{c_i}{v_i^2} \frac{f_i^2}{f_j^2} \theta_1 \cos \theta_i \cos^2 \theta_i.
\]  \hspace{1cm} (14)

Here \( \theta_i \) is the angle between the vertical and the infrasonic wave direction, \( c \) is the light velocity, and \( v_i \) is the infrasonic velocity.

3. Calculation results

The given relation (14) between the frequencies of the infrasonic waves, \( f_i \), generated during various atmospheric disturbances on and under the ground and measured experimentally, allows us to calculate the whistler frequencies \( f_j \) which, as shown above, can be generated or enhanced in the given case in the ionospheric E-region.

Table 1. Frequencies \( f_j \) (in kHz) of whistlers generated by infrasound in ionospheric E-region

<table>
<thead>
<tr>
<th>( f_i ), Hz</th>
<th>0.05</th>
<th>0.1</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( \theta_i = 0^\circ )</td>
<td>0.130</td>
<td>0.520</td>
<td>52</td>
<td>208</td>
</tr>
<tr>
<td>5</td>
<td>( \theta_i = 30^\circ )</td>
<td>0.098</td>
<td>0.450</td>
<td>39</td>
<td>156</td>
</tr>
<tr>
<td>10</td>
<td>( \theta_i = 0^\circ )</td>
<td>0.047</td>
<td>0.187</td>
<td>18.72</td>
<td>74.88</td>
</tr>
<tr>
<td>10</td>
<td>( \theta_i = 30^\circ )</td>
<td>0.035</td>
<td>0.140</td>
<td>14.04</td>
<td>56.16</td>
</tr>
</tbody>
</table>

Note that \( f_j \) depends not only on \( f_i \) but on the electron density \( N \) in the ionospheric E-region as well, since \( f_j = f_i (N) \). The table presents \( f_j \) results for both cases: a) \( \theta = 0^\circ \), \( \theta_i = 0^\circ \); b) \( \theta = 0^\circ \), \( \theta_i = 30^\circ \); \( v_i = 500 \text{ m/s} \), \( n_e = 8 \times 10^8 \text{ s}^{-1} \), \( f_i \) - results being taken from the papers on observing infrasound during thunderstorms, earthquakes and explosions (see, for example, Al'perovich et al. (1983); Baker and Cotten (1971); Davies and Baker (1965); Grigor'ev and Dokuchayev (1981); Rai and Kisabeth (1967)).

The results have shown that the \( f_i \) whistler frequencies change from hundreds of Hz to hundreds of kHz for 0.05 Hz < \( f_i < 5 \text{ Hz} \). These calculations are confirmed by the experimental data taken from the references for the case of discovering enhancements of the whistlers in the epicenter earthquake zone, in the region of strong thunderstorms, etc (see, for example, Mikhailova et al. (1991); Mikhailov et al. (1997)).

It should be noted that though infrasonic waves of \( f_i > 5 \text{ Hz} \) reach the heights of the ionospheric part investigated, we do not consider them because of their strong fading. It might be also believed that the mechanism considered here will be valid for the heights range of 170–220 km reached by the infrasonic waves with \( f_i < 1 \text{ Hz} \), where the electron density is higher than that in the E-region. Note that low frequency whistlers seem to be recorded near the disturbance source epicenter since, as shown in Mazur (1988), the presence of a rather powerful layer of Pedersen conductivity leads to impossibility for a low frequency whistler to
propagate along the Earth surface (in the ionospheric region considered) over long distances (more than hundreds of kilometers). Nevertheless, they can be enhanced rather strongly since for low frequency whistlers the ionosphere may serve as a fairly good resonator with the quality factor $Q_w = \pi n \left( \frac{2 \sigma_{\parallel}}{\sigma_{\perp}} \right)^{1/2} \left( \cos \theta \right)^{1/2} \left( 1 + \cos \theta \right)^{-1}$ (Mazur (1988)). For the harmonics with $n \sim 3-5$ and the calculated $\sigma_{\parallel}/\sigma_{\perp} \approx 2$ (Volker and Carpenter (1975)) ($\sigma_{\parallel}, \sigma_{\perp}$ are the Hall and Pedersen conductivities), it may be reached $Q_w \approx 20-30$. Such oscillations can be recorded, for instance, in natural radio noise spectra.

4. Conclusion

It is shown that infrasonic waves generated in the atmosphere during disturbances of different natures and propagating in the Earth’s ionosphere can generate or enhance low frequency whistlers in the ionospheric E-region. From the dispersion equation it is derived a relation between frequencies of infrasound and those of a whistler; using papers dealing with experimental data on an interval of frequencies of infrasound generated in the atmosphere during disturbances of different natures, a frequency interval, in which generation of whistlers is possible, is calculated.

References


(Received June 28, 1999; revised August 9, 1999; accepted September 16, 1999)