Modeling Internationally Diversified Investment Portfolio

Natalia Chernova, Maryna Mashchenko
Simon Kuznets Kharkiv National University of Economics
Kharkiv, Ukraine
natacherchum@gmail.com,
mmashchenko@ukr.net

Olena Sergienko, Oleksandr Bilotserkivskyi, Olena Shapran
National Technical University «Kharkiv Polytechnic Institute»
Kharkiv, Ukraine
serhelenka@gmail.com, abelocerk@gmail.com,
elena.shapran82@gmail.com

***Abstract*—A diversified portfolio can deliver improved performance and lessened risk relative to not diversified one. The diversification may be achieved in different ways: an investor may allocate money in different asset classes; or in stocks of different industries; or in bonds with different maturities; or in stocks with large, middle and small market capitalization; or in different commodities and so on. Another way is international (global) diversification. The paper aim is to form an optimal investment portfolio containing assets of different countries. To achieve this the following tasks should be solved: carry out a preliminary financial analysis of assets (stock indices) of different countries and determine the set of assets that are eligible for investment; form an optimal portfolio of selected stock indices; carry out a comparative analysis of the efficiency of the obtained portfolio and the benchmark portfolio, which is domestically oriented. The proposed algorithm for modeling internationally diversified optimal portfolio includes the following core steps: forming the information base of the research; classification of assets into homogeneous groups and detecting the group that is suitable for the investment; modeling diversified portfolio and a benchmark portfolio; comparison of obtained portfolios performance.**

Keywords—stock index; investment; portfolio; diversification; model; cluster; financial analysis.

I. INTRODUCTION

It goes without saying that diversification is "the only 'free lunch' in finance." H. Markowitz was the first who argued that a diversified portfolio can deliver improved performance and lessened risk relative to not diversified one. The key assumption of his theory is that diversified portfolio must consist of non-correlated assets. To form an appropriate basket an investor may allocate money in different asset classes, e. g. in stocks, bonds, currencies and precious metals; or in stocks of different industries; or in bonds with different maturities; or in stocks with large, middle and small market capitalization; or in different commodities and so on. In addition to the methods listed above, another one is international (global) diversification. It means that there are not only local but also international assets in the portfolio. Through international diversification, using the assets of countries whose economic cycles may occur with some lag, the investor can significantly reduce portfolio risk.

In general, there are several ways to invest internationally. Firstly, you may directly purchase foreign stocks on foreign exchanges, or you may purchase depository receipts on foreign stocks on local exchanges. Secondly, you may invest through exchange-traded funds (ETFs) or mutual funds that hold a basket stocks and bonds of a certain country Thirdly, you may invest in country economy through a particular stock index derivatives.

Developing this idea, you can form a portfolio of several stock indices belonging to different countries. This helps to achieve the effect of diversification by geographical and sectoral characteristics, as well as by the level of capitalization. In general, the idea of international diversification of investment portfolios is found in the works of many authors [1-9]. It is proved that the distribution of funds between the assets of domestic companies and foreign assets often reduces the overall risk of the portfolio. A comparative analysis of investment strategies in developed and emerging markets is conducted in [1, 4, 5]. In [3] it was proved that for some international portfolios the degree of diversification can be significantly increased both by adding a component of emerging markets and by increasing the share of instruments in developed markets. It should be noted that most of the above studies solve the problem of increasing the level of diversification of the already formed portfolio. Typically, some underlying portfolio is considered, which contains the assets of domestic companies within one country, and then research is conducted to increase its efficiency by including foreign international assets. Therefore, the task of forming a portfolio, which a priori should contain assets from different countries, represented only by stock indices, is quite relevant. The idea of investing in the country by investing not in a certain set of shares issued by different companies, but in a single stock index, is very promising. In carrying out such an operation, the investor simultaneously monitors the trends of the entire economic system, rather than its individual sectors or elements. Moreover, in this case it is not necessary to determine the relative shares of different sectors of the economy or individual companies in the portfolio, because stock indices are already formed as weighted averages, usually taking into account the level of capitalization of their components.

Assets from other countries tend to perform a little differently and typically balance out a domestic-heavy investment portfolio nicely. However, global equity return correlations have experienced a significant increase resulting from trade and financial globalization. That is why the topic of the study, aimed to form an optimal

investment portfolio containing assets of different countries, is of great scientific and applied interest.

II. Literature Review

Investment strategy - a strategy for allocating capital among different categories to gain profit [10-14]. To form the initial set of assets an investor firstly should apply fundamental analysis techniques, that help to evaluate the intrinsic value of an asset and analyze the factors that could influence its price in the future. This form of analysis includes estimations based on many components related to stocks, including financial statements, news releases, domestic political conditions, trade agreements and external politics, competitor analysis.

The investment strategy should take into account the investor's risk tolerance as well as future needs for capital. Risk tolerance is the amount of risk that an investor is able to handle. The rate of return information can be used to help the investor decide upon the types of investments to engage in and the level of risk to take on. The investment strategy can be applied on the basis of so cold portfolio theory - the theory of investment management, based on statistical methods for optimizing the portfolio structure according to the selected criterion for the ratio of profitability and risk.

H. Markowitz was the first who formulate the portfolio theory core principals. He offered a mathematical model for the formation of an optimal portfolio of securities, and also gives methods for constructing such portfolios under certain conditions. Instead of focusing on the risk of each individual asset, he demonstrated that a diversified portfolio is less volatile than the total sum of its individual parts. While each asset itself might be quite volatile, the volatility of the entire portfolio can actually be quite low. The problem is solved by quadratic optimization methods. The only problem here is that these methods are applicable only for comparatively low dimensional tasks [11].

While Markowitz suggested to form optimal portfolio of stocks only, J. Tobin later proposed to include risk-free assets (government bonds) in the initial set of securities. In fact, his approach is macroeconomic, since in this case the main object of study is the distribution of total capital into two forms: cash and non-cash [14].

The main result of capital asset pricing model (CAPM) was the establishment of a ratio between profitability and asset risk for market equilibrium. It was postulated that risk of any asset consists of two parts – non-systematic and systematic. When choosing an optimal portfolio investor should take into account only systematic or non-diversifiable risk [13].

III. Aims and Methods

The aim of the work is to form an optimal investment portfolio containing assets of different countries. To achieve this the following tasks should be solved:

- carry out a preliminary financial analysis of assets (stock indices) of different countries and determine the set of assets that are eligible for investment;
- form an optimal portfolio of selected stock indices;
- carry out a comparative analysis of the efficiency of the obtained portfolio and the benchmark portfolio, which is domestically oriented.

The proposed algorithm for modeling internationally diversified optimal portfolio includes the following core steps:

- Step 1. Forming the information base of the research.
- Step 2. Classification of assets into homogeneous groups and detecting the group that is suitable for the investment.
- Step 3. Modeling diversified portfolio and a benchmark portfolio.
- Step 4. Comparison of two portfolios.

Let’s consider these steps in detail.

Step 1. Stock investing requires careful analysis of financial data to find out the asset's true worth. This is generally done by analyzing its financial ratios, which may be divided into following groups:

- Profitability ratios;
- Liquidity ratios;
- Solvency (leverage) ratios;
- Valuation ratios.

Liquidity ratios show a company's ability to meet its near-term obligations, and it is a major measure of financial health. The higher the ratio, the more liquid a company is. Some of the key liquidity ratios include: current ratio, quick ratio, cash ratio, cash conversion cycle, operating cash flow ratio, receivables turnover, inventory turnover, working capital turnover.

Profitability ratios are used to measure and evaluate the ability of a company to generate income (profit) relative to revenue, balance sheet assets, operating costs, and shareholders’ equity during a specific period of time. They show how well a company utilizes its assets to produce profit and value to shareholders. High profitability ratios usually mean that the business is doing well, generating revenue, profit and cash flow. Some key profitability ratios are: gross margin, operating margin, net profit margin, earnings before interest, taxes, depreciation and amortization (EBITDA) margin, operating cash flow margin, return on assets, return on equity, return on invested capital, return on investment.

Solvency ratios, also known as leverage ratios, are used by investors to get a picture of how well a company can deal with its long-term financial obligations. They determine the chances of the company's long-term survival. Some of the most popular solvency ratios include: debt to total assets, debt to equity, time interest earned, interest coverage ratio, net income to liabilities, times interest earned.

Valuation ratios put that insight into the context of a company’s share price, where they serve as useful tools for evaluating investment potential. In general, the lower the ratio level, the more attractive an investment in a company becomes. These ratios are: price-to-earnings, price-to-book, price-to-sales, price-to-cash flow, price/earnings-to-growth,

dividend yield, enterprise value/earnings before interest, taxes, depreciation and amortization ratio.

Step 2. The initial set of analyzed companies should be divided into homogeneous groups so that elements belonging to one group are located relatively close to each other in a multidimensional space of indicators, and elements that fall into different groups are characterized by relatively large values of the chosen similarity measure. We will solve the described problem using k-means clustering algorithm. The core steps of the algorithm are the following:

1) Choose the number of clusters k

2) Select k random points from the data as centroids

3) Assign all the points to the closest cluster centroid

4) Recalculate the centroids of newly formed clusters

5) Repeat steps 3 and 4 until centroids of newly formed clusters do not change

Step 3. The classical Markowitz model will be applied to obtain the optimal portfolio structure. The model takes into consideration the following basic assumptions: investors are rational and seek to maximize the expected return; investors are risk averse so they require a higher expected return to compensate for higher risk accepted; investors rely merely on expected returns and variance to make investment decisions; investors cannot influence prices; risk is estimated as the standard deviation of return.

Portfolio return is calculated as the weighted average sum of the returns of individual securities:

$$R_p = \sum w ,$$

where $\sum$ - weight of the i-th instrument; r_i - return of the i-th instrument.

Portfolio risk is calculated as following:

$$\delta_p^2 = \sum_i w_i^2 \delta_i^2 + \sum_i \sum_{i \neq i} w_i w_j \delta$$

where δ_i - risk of the i-th instrument; ν_{ij} - the correlation coefficient between the returns on instruments i and j.

The model allows to obtain the so-cold efficient frontier - a set of portfolios that give us the highest return for the lowest possible risk.

Step 4. Portfolio comparison is conducted according to risk and return criteria.

IV. Findings

Let's consider the applications of the algorithm.

On the first step the initial data set of financial ratios for seventeen world stock indexes was formed [15]. The list of stock indices is presented in the Table 1. We use the following financial ratios:

- Current ratio (CUR_RATIO);
- Debt to total equity (TOT_DEBT_TO_TOT_EQY);
- Debt to total assets (TOT_DEBT_TO_TOT_ASSETS);
- Dividend yield(EQY_DVD_YLD_EST);
- Price to earnings (PE_R)ATIO);
- Enterprise value to Earnings before Interest, Taxes, Depreciation and Amortization (EV/EBITDA);
- Price-to-book (PRICE_BOOK_VAL);
- Return on equity (RETURN_COM_EQY);
- Return on assets (RETURN_ON_ASSET);
- Operating Return on equity margin (OPER_MARGIN);
- Return on invested capital (RETURN_ON_CAP).

TABLE I. List of stock indices

Index Name	Country	Symbol
DAX Index	Germany	DAX
NIKKEI Index	Japan	NKY
CAC 40 index	France	CAC
SMI Index	Switzerland	SMI
AEX Index	Netherlands	AEX
BEL 20 Index	Belgium	BEL20
Tel Aviv Stock Exchange 35 Index	Israel	TA-35
S&P/TSX Composite Index	Canada	SPTSX
OMX Stockholm 30 Index	Sweden	OMXS30B
Korea Stock Exchange KOSPI Index	Korea	KOSPI
Ibovespa Brasil Sao Paulo Stock Exchange Index	Brasil	IBOV
RTS Index	Russia	RTSI$
S&P/BMV IPC	Mexica	MEXBOL
S&P/CLX IPSA CLP TR	Chile)	IPSA
S&P BSE SENSEX Index	India	SENSEX
Borsa Istanbul 100 Index	Turkey	XU100
Shanghai Shenzhen CSI 300 Index	China	SHSZ300

According to the Step 2 we need to determine the number of clusters to be formed within the classification model. We applied the hierarchical clustering algorithm to choose the appropriate number of clusters. The obtained dendrogram is presented in Fig. 1.

According to the obtained results the decision was made to apply k=3 as input parameter for k-means clustering.

According to the analysis of variance results the indicator of current ratio (CUR_RATIO) must be excluded from the research because its significance level exceeds 50%. All further calculations were conducted without it. Table 2 presents the final analysis of variance results. It can be seen that for eight indicators the significance levels do

not exceed 5%, and only for two of them the levels are between 5% and 10% (PE_RATIO and EV/EBITDA).

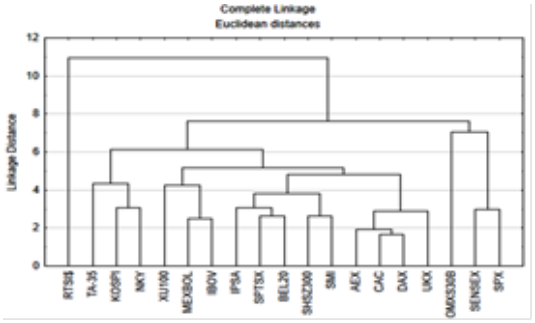

Fig. 1. Classification Dendrogram

TABLE II. ANALYSIS OF VARIANCE

Variable	Between SS	Within SS	p-level
TOT_DEBT_TO_TOT_EQY	6.328	11.672	0.0312
TOT_DEBT_TO_TOT_ASSETS	5.762	11.880	0.0422
EQY_DVD_YLD_EST	9.482	8.517	0.0025
PE_RATIO	5.587	12.412	0.0511
EV/EBITDA	4.800	13.199	0.0835
PRICE_BOOK_VAL	9.685	8.314	0.0020
RETURN_COM_EQY	11.685	6.314	0.0002
RETURN_ON_ASSET	11.942	6.057	0.0001
OPER_MARGIN	8.920	9.079	0.0041
RETURN_ON_CAP	10.072	7.927	0.0014

Fig. 2 shows the graphs of means for obtained clusters. First of all, you need to pay attention to Cluster 3. Most of its average values demonstrate extreme values. It has the highest dividend yield, the lowest level of debt along with the highest profitability. The lowest values of PE_RATIO and EV/EBITDA indicate that assets from this cluster are undervalued. The cluster consists of only one participant – RTSI$.

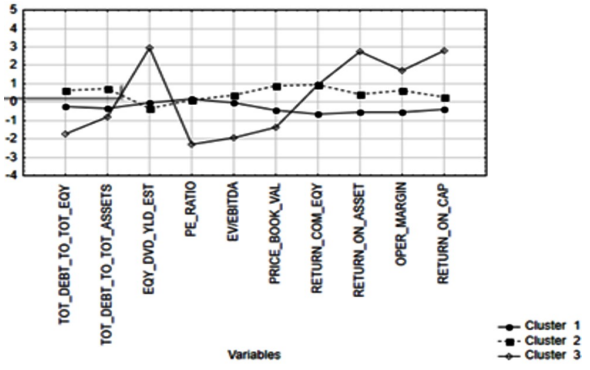

Fig. 2. Cluster Means

Cluster 1 has rather low debt levels according with low levels of such valuation ratios as EV/EBITDA and PRICE_BOOK_VAL. We indicate assets from this cluster as undervalued too. The cluster contains the following indexes as UKX, DAX, NKY, CAC, AEX, BEL20, TA-35, SPTSX, KOSPI, IPSA, XU100.

As for Cluster 2 we see here the highest debt levels among with low dividend yields and high valuation ratios. We indicate this cluster as being overvalued at the moment.

So we suggest to apply assets from Cluster 1 and Cluster 3 as inputs for Markowitz optimization model.

Step 3. The input data set consists of daily closing prices for time period 2019-2020. Fig. 3 presents stock indices in two-dimensional "risk-return" space. Three objects have demonstrated negative returns within the observed period – they are IPSA, UKX and TA-35. They must be excluded from the research because the classical Markowitz model assumes only positive returns (and only long positions in the portfolio). The other indices show differences in risk and return values, so the initial set of financial instruments is rather diversified according to these measures.

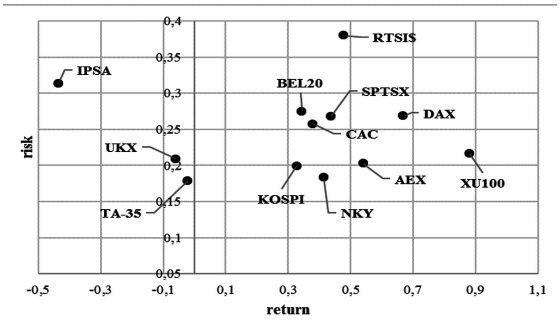

Fig. 3. Risk and return levels

Fig. 4 presents ten portfolios within obtained efficient frontier.

Asset	Portfolio							
	P1	P2	P3	P4	P5	P6	P7	P8
RTS	3%	1%	0%	0%	0%	0%	0%	0%
DAX	0%	0%	0%	0%	0%	7%	7%	7%
NKY	34%	34%	33%	33%	32%	29%	21%	13%
AEX	8%	10%	12%	12%	10%	0%	0%	0%
SPTSX	9%	6%	3%	0%	0%	0%	0%	0%

Fig. 4. Asset allocations in the optimal portfolios

Two assets have not participated in any of ten portfolios – CAC and BEL20. According to Fig. 3 they have rather high risk levels and low returns. RTSI$ was included only in P1 and P2 with very low percent. This fact may be explained by the highest level of risk.

Risk and return levels are increased in direction from P1 to P10. The riskiest portfolio includes the only participant XU100. This asset is the only one which presented in every portfolio. Its share is increasing simultaneously with the portfolio risk growth.

The second active participant is NKY. Its share is changing slightly in the first five portfolios and then drops sharply in the last five portfolios. In P6-P9 it is partially replaced by DAX.

The initial set of stock indices in P1-P10 consists of four elements from the developed sector (DAX, NKY, AEX, SPTSX) and three elements from the emerging sector (RTSI$, KOSPI, XU100). Fig. 5 presents portfolio distribution between mentioned sectors. The developed sector share changes in descending order from 51% in P1 to 43% in P5 and then drops rapidly to zero level in P10. Such developed indices as AEX and SPTSX are present in only first five portfolios with relatively low risk levels.

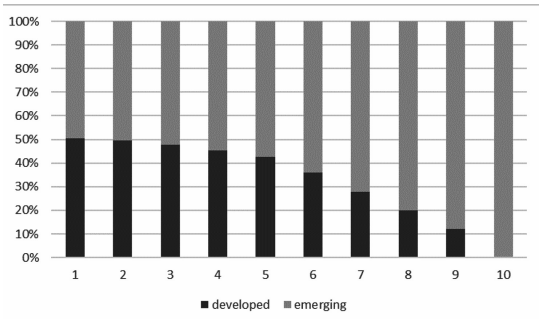

Fig. 5. Asset allocation between developed and emerging sector

Fig. 6 shows the efficient frontier graph. The seventh portfolio (P7) is characterized with the highest Sharp Ratio. P7 consists of DAX (7%), NKY (21%) AND XU100(72%).

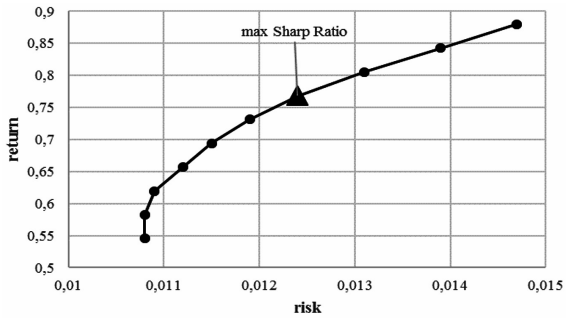

Fig. 6. Efficient frontier

V. Conclusions

If take P7 and try to invest separately in each of its components, we obtain Sharp Ratio of 1.56, 1.06 and 1.63 respectively for DAX, NKY and XU100. For the weighted P7 the appropriate value is 1.64. We see a valuable effect of diversification for developed countries. As it was noted earlier, in recent years developed sector produce comparatively lower returns. That is why developed portfolio performance may be improved through inclusion of the emerging component. As for emerging sector as domestic part of the portfolio we see that the diversification effect may not be so valuable. In case of XU100 Index we see the highest return levels among with moderate risks. That is why its performance was not improved significantly in P7. From the other hand, those return and risk levels may be inherent only for the analyzed period 2019-2020.

The future researches should take into consideration retrospective periods of larger length, especially those ones which are indicated by economic expert's community as crisis periods. The separate models should be constructed for so-cold defensive portfolios and growth portfolios, which a priori have different goals and different diversification parameters.

Step 3 of the proposed algorithm is based on the assumption that assets within the initial set must have only positive returns and must have only long positions

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