Helical tractrices and pseudo-spherical submanifolds in $\mathbb{R}^n$

K. Stiepanova[1], V. Gorkavyy[2]

[1]*S. Kuznets Kharkiv National University of Economics, Kharkiv, 61166, Ukraine*

[2]*B. Verkin Institute for Low Temperature Physics and Engineering of, Ukraine National Academy of Sciences, Kharkiv, 61123, Ukraine*

The classical theory of pseudo-spherical submanifolds and their Bianchi-Bäcklund transformations deals with n-dimensional submanifolds in $(2n-1)$-dimensional spaces of constant curvatures, see [1]-[2]. In [3] Yu. Aminov and A. Sym settled the problem asking for generalizations of the classical theory to the case of pseudo-spherical submanifolds with arbitrary codimension. A survey of results concerning this problem may be found in [4].

Particularly, we are interested in searching for submanifolds with arbitrary dimension and codimension, which may be viewed as analogues / generalizations of the classical Beltrami and Dini surfaces in $\mathbb{R}^3$.

The pseudo-spherical submanifolds in $\mathbb{R}^n$ that admit Bianchi transformations degenerated to curves and hence inherit features of the Beltrami surface, were completely described in [5]. As well, the pseudo-spherical submanifolds in $\mathbb{R}^n$ that admit Bäcklund transformations degenerated to straight lines and hence inherit features of the Dini surfaces, were completely described in [6].

For constructing submanifolds in $\mathbb{R}^n$ that admit Bäcklund transformations degenerated to curves different from straight lines, we propose to use *helical tractrices*. By definition, a smooth oriented curve γ in $\mathbb{R}^n$ is called a helical tractrix if the endpoints of unit segments tangent to γ form a curve of constant curvatures in $\mathbb{R}^n$.

It is conjectured that submanifolds in $\mathbb{R}^n$ obtained by particular skew rotations of helical tractrices have constant negative sectional curvature and inherit basic features of the Dini surfaces concerning their Bäcklund transformations. We provide arguments partially confirming this conjecture.

1. Yu.A. Aminov, *Geometry of submanifolds.* CRC Press, London, 2001.
2. K. Tenenblat, *Transformations of manifolds and applications to differential equations.* Longman, Harlow, 1998.
3. Yu. Aminov, A. Sym, *On Bianchi and Backlund transformations of surfaces in* E^4, Math. Phys., Anal., Geom., **3** (2000), 505-512.

4. V. Gorkavyy, *Generalization of the Bianchi-Bäcklund Transformation of Pseudo-Spherical Surfaces*, J. of Math. Sciences, **207** (2015), 467-484.
5. V. Gorkavyy, O. Nevmerzhitska, *Pseudo-spherical submanifolds with a degenerate Bianchi transformation*, Res. in Math., **60** (2011), 103-116.
6. V. Gorkavyy, O. Nevmerzhitska, *Degenerate Bäcklund transformation*, Ukrainian Mathematical Journal, **68** (2016), 41-56.

A Steiner formula in the L_p Brunn Minkowski theory

K. Tatarko[1], E. Werner[2]
[1] *University of Alberta, Edmonton, Canada*
[2] *Case Western Reserve University, Cleveland, USA*

The classical Steiner formula is one of the central parts of Brunn Minkowski theory. It expresses the volume of the parallel body $K + tB_2^n$ of a convex body K with Euclidean ball B_2^n as a polynomial in parameter t, where the intrinsic volumes arise as the coefficients of this polynomial.

The L_p Brunn Minkowski theory is an extension of the classical Brunn Minkowski theory which was initiated by Lutwak in [1] and rapidly evolved over the past years. It centers around the study of affine invariants associated with convex bodies. One of the main objects in the L_p Brunn Minkowski theory is the L_p affine surface area

$$as_p(K) = \int_{\partial K} \frac{H_{n-1}(x)^{\frac{p}{n+p}}}{\langle x, \nu(x) \rangle^{\frac{n(p-1)}{n+p}}} d\mathcal{H}^{n-1}(x),$$

where $\nu(x)$ denotes the outer unit normal at $x \in \partial K$, the boundary of K, $H_{n-1}(x)$ is the Gauss curvature at x and $\mathcal{H}^{n-1}$ is the standard surface area measure on ∂K.

In this talk we present an analogue of the classical Steiner formula for the L_p affine surface area of a Minkowski outer parallel body for any real parameter p. This new Steiner type formula includes the classical Steiner formula and the Steiner formula from the dual L_p Brunn Minkowski theory as special cases.We introduce the coefficients in our new Steiner type formula which we call L_p quermassintegrals and also observe some of their properties.

1. E. Lutwak, *The Brunn-Minkowski-Firey theory II. Affine and geominimal surface areas*, Adv. Math. **118** (2) (1996), 244-294.