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MULTI-CRITERIA CHOICE OF ALTERNATIVES UNDER FUZZY INFORMATION

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ABSTRACT. ***The solution of the problem of multicriteria alternative choice in the conditions of fuzzy expert assessments of compliance with alternatives to the requirements of criteria is described. We consider the situation when the rating of the alternatives can comprise several experts that form an individual system of criteria for evaluation of alternatives using numerical or linguistic form submissions. The construction of a unified system of criteria is proposed to perform on the basis of assessing the truth of linguistic statements about the feasibility of the inclusion of criteria in a unified system, which subsequently identifies non-overlapping subsets of criteria that characterize the opportunities and threats in the implementation of alternatives. In contrast to the known studies, the final choice of the best alternative is based on the ratio of integral estimates according to the criteria characterizing opportunities and threats.***

KEYWORDS: fuzzy sets, multicriteria choice, linguistic values, evaluation criteria, fuzzy mapping.

JEL classification: C02, C18, D81, O22.

Introduction

In the management activities of various organizations, enterprises, firms often encountered the problem of choosing a decision from the many possible (Andriosopoulos et al., 2012; De Felice, Petrillo, 2013; Duvivier et al., 2013; Pech, 2010; Thery, Zarate, 2009; Yu et al., 2009; Zopounidis, Doumpos, 2013). This is, for example, the formation of packages of projects for the economic development program (Chui, Chan, 1994; Rojas-Zerpa, Yusta, 2015; Schafer, Gallemore, 2016), the choice of a corporate information system for enterprise informatization (Camara, Cabral, 2014; Nazari-Shirkouhi et al., 2011), preparation for a decision on new products (Mastorakis, Siskos, 2016; Poveda-Bautista, 2016; Sielska, 2015, etc.)

Various methods, models and algorithms of multicriteria choice in various economic problems were actively discussed and described in the scientific literature. However, the approach using fuzzy modelling for such problems have been described not enough (Chakhar et al., 2016; Durbach, Calder, 2016; Thomaidis et al., 2006; Ye, 2010).

Despite the differences in these problems they are united by a number of provisions.

In the first, all the problems are the tasks of multi-criteria alternative choice.

Secondly, in the decision may be involved several expert groups formed by employees of the organizations or enterprises, independent experts, and experts representing concrete projects. Thirdly, the decision is carried out under conditions of uncertainty regarding both the criteria and estimates of criteria compliance. At that, the process of their formation has an expert character, and expert estimates a fundamentally characterized by uncertainty.

Accordingly, is arise specific sequence of applied problems of mathematical modelling.

First of it is the choice of the mathematical apparatus for solving the problems of an alternative multi-criteria selection, capable of operating in conditions of non-statistical uncertainty and in the presence of estimates both in numeric or verbal form. Conditions of non-statistical uncertainty related to the fact that the problem, usually have a unique character. And some of the estimates not have or cannot be represented in numerical form.

The second task is matching of expert assessments. Wherein selection and application of known matching techniques can be difficult due to limitations on the number of involved experts. Furthermore, is reasonably practicable that algorithm of expert estimates matching would be in the framework of selected mathematical apparatus.

Therefore, the development (based on the selected mathematical apparatus) of algorithmic instruments for matching expert estimations and structuring alternatives suitable for computer implementation is needed.

The fuzzy set theory is the most appropriate in a logical and mathematical point of view to solving the problems that can be justified by the following circumstances.

Theory of fuzzy sets created specifically for presentation in a strict mathematical form of indefinite judgments inherent in human nature. Numerical evaluations of the criterion of compliance in the form of fuzzy numbers, in contrast to the commonly used points (scores or grades), present expert opinions in the form of intervals of values and their capabilities distribution defined by the membership function.

By methods of fuzzy set theory can be solved the problem of coordination (harmonizing) of expert evaluations, even in the absence of restrictions on the number of involved experts and analysed criteria. It is known, for example, that the value of concordance factor used to estimate the coherence expert judgment depends on the number of experts. Theory of fuzzy sets has a large set of algorithmic methods for solving the problem of multi-criteria alternative choice.

1. Formulation of the Problem

Suppose we have a lot of projects (alternatives) that are planned for implementation - $P=\{P_i\ ;i\ =\overline{1,I}\ \}$. For making a decision on the selection of projects must be developed a system of evaluation criteria, the creation of which may be involved different groups of experts who act independently.

Assume for simplicity that it is internal experts (employees of organization or company) and external experts. Estimates of criteria compliance may have both numerical and verbal form (Roy, Slowinski, 2013). Thus, to evaluate alternatives finite sets of both quantitative and qualitative criteria must be drawn up - $C_S=\{c_s:s=\overline{1,S}\}$, built by internal experts - $C_R=(c_r:r=\overline{1,R})$, proposed by external experts.

In general, combining a set of evaluation criteria can be constructed as a logical sum:

$$\hat{C}=C_S\oplus C_R=(C_S\cap C_R)\cup(C_S\cup C_R) \qquad (1)$$

Then, to solve the problem of choosing the best alternative, it is necessary to construct a mapping:

$$G:P\rightarrow\hat{C} \qquad (2)$$

Considering that the mapping (2) is fuzzy (that is, to specify unequivocal, strict conformity of projects to requirements of the criteria is impossible), mapping (2) can be rewritten as follows:

$$\tilde{G} : P \xrightarrow{\mu} \hat{C}, \qquad (3)$$

where $\tilde{G}$ - fuzzy mapping; μ- degree of mapping fulfillment $\tilde{G}$.

Building a combined set of criteria for the relation (1) is permissible if it is assumed that all the criteria are appropriate to include in a set. Otherwise, such a decision can't be considered as correct.

2. Procedure for Forming a Set of Evaluation Criteria

It is proposed the following procedure for constructing a combined set of evaluation criteria. The advisability of including a specific criterion in the agreed system will be evaluated in verbal form, i.e. by construction term set $T = \{\tau_k : k = \overline{1, K}\}$ (for example < inadvisable, advisable, very advisable >, K=3). Note that the dimension of the term set does not affect the generality of the decision procedure. Of course, appropriateness of inclusion of criteria in the combined system can be expressed in numerical form, for example as coordinates of the eigenvectors of a matrix of pairwise comparisons. However, it will require quite cumbersome calculations. In addition, it may be that not all of the involved experts possess this method.

The verbal representation of the advisability of including evaluation criteria in the combined system is more simple. Should be noted that to this work may be involved an additional group of independent experts, which will be engaged only in the task of building a combined system of criteria.

Experts involved in the evaluation of the feasibility of the inclusion criteria in a combined system, have to build a mapping,

$$G_c : C \to T \qquad (4)$$

for both sets of elements C_s and C_R. For example, when the number of experts is five, term set contains three elements, a mapping (4) can be defined by *Table 1* and *Table 2*.

Table 1. Assessing the feasibility of the inclusion criteria of sets *Cs* in the combined system

E_n /N	Term set	Criterion number									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
N1	inadvisable	0	0	0	0	0	0	1	0	0	0
	advisable	0	0	1	0	0	0	0	1	1	0
	very advisable	1	1	0	1	1	1	0	0	0	1
N2	inadvisable	0	0	0	0	0	0	1	0	0	0
	advisable	1	0	1	0	1	0	0	0	1	0
	very advisable	0	1	0	1	0	1	0	1	0	1
N3	inadvisable	0	0	0	0	0	0	0	0	0	0
	advisable	0	0	0	0	1	0	1	0	1	0
	very advisable	1	1	1	1	0	1	0	1	0	1
N4	inadvisable	0	0	0	0	0	0	1	0	0	0
	advisable	0	0	1	0	0	0	0	0	0	0
	very advisable	1	1	0	1	1	1	0	1	1	1
N5	inadvisable	0	0	0	0	0	0	1	0	0	0
	advisable	1	0	0	0	0	0	0	1	1	0
	very advisable	0	1	1	1	1	1	0	0	0	1

Source: own calculations.

Table 2. Assessing the feasibility of the inclusion criteria of sets C_R in the combined system

E_n/N	Term set	1	2	3	4	5	6	7	8
N1	inadvisable	1	0	0	0	0	1	0	0
	advisable	0	0	1	0	0	0	0	1
	very advisable	0	1	0	1	1	0	1	0
N2	inadvisable	0	0	0	0	0	1	0	0
	advisable	1	0	0	1	0	0	0	0
	very advisable	0	1	1	0	1	0	1	1
N3	inadvisable	1	0	0	0	0	0	0	0
	advisable	0	1	0	1	0	0	1	0
	very advisable	0	0	1	0	1	0	1	1
N4	inadvisable	1	0	0	0	0	1	0	0
	advisable	0	1	1	1	0	0	0	0
	very advisable	0	0	0	0	1	0	1	1
N5	inadvisable	0	1	0	0	0	1	0	0
	advisable	1	0	0	1	0	0	1	1
	very advisable	0	0	1	0	1	0	0	0

Source: own calculations.

To make a decision on the inclusion of criteria in the combined system let us introduce evaluation $\mu_{kj} = \min(1, \frac{1}{N}\sum_{i=1}^{N} w_{ik}), k \in [1, K], j \in [1, J],$. The results of calculations of this assessment for *Cs*, C_R criteria are presented in *Table 3* and *Table 4* respectively

From *Table 3* it follows that criterion $c_7 \in C_S$ received the highest estimate for the linguistic meaning “inadvisable” and it should not be included in the combined criteria system.

Table 3. Results of processing of expert opinions on the criteria C_S

Criterion / Term	1	2	3	4	5	6	7	8	9	10
inadvisable	0	0	0	0	0	0	4	0	0	0
$\mu_{1,j}$	0	0	0	0	0	0	0.8	0	0	0
advisable	2	0	3	0	2	0	1	2	4	0
$\mu_{2,j}$	0.4	0	0.6	0	0.4	0	0.2	0.4	0.8	0
very advisable	3	5	2	5	3	5	0	3	1	5
$\mu_{3,j}$	0.6	1	0.4	1	0.6	1	0	0.6	0.2	1

Source: own calculations.

Table 4. Results of processing of expert opinions on the criteria C_R

Criterion / Term	1	2	3	4	5	6	7	8
inadvisable	3	1	0	0	0	4	0	0
μ_{1j}	0.6	0.2	0	0	0	0.8	0	0
advisable	2	2	3	4	0	1	1	3
μ_{2j}	0.4	0.4	0.6	0.8	0	0.2	0.2	0.6
very advisable	0	2	2	1	5	0	4	2
μ_{3j}	0	0.4	0.4	0.2	1	1	0.8	0.4

Source: own calculations.

A similar situation occurs for the criteria $c_1 \in C_R$, $c_6 \in C_R$. As a result, we obtain new sets $\hat{C}_S$ and $\hat{C}_R$ and the combined system of criteria

$$\hat{C} = (\hat{C}_S \cup \hat{C}_R) \cup (\hat{C}_S \cap \hat{C}_R) = (\hat{c}_n : n = \overline{1:N}), \qquad (5)$$

where N- the number of criteria in the combined system.

Because the single criteria from the sets $\hat{C}_S$ and $\hat{C}_R$ can coincide, then the dimension of the united set may be less than the sum of separate dimensions of sets incoming therein, i.e. $N \leq R+S$.

3. Assessment of Conformity Alternatives to Criteria Requirements

After constructing a system (5) is necessary to carry out conformity assessment of alternatives to the criteria requirements by construction a mapping (3).

In this case are possible various variants, defined by conditions that must be agreed upon at the beginning of solving the problem.

It is necessary to determine, whether be assumed equivalence of criteria, or they have a variety weights, in which scale is presented assessments of criterion conformity.

Suppose that criteria included in the system (5) have different weights (various importance), accordingly, it is necessary to solve the problem of their appointment.

For this purpose, the most appropriate to use the matrix of pairwise comparisons with subsequent calculation of the coordinates of the eigenvector, which can be interpreted as the weights of corresponding criterion.

As grounds for this proposal, we can indicate the possibility of monitoring the correctness of the obtained values of the weights using coherence index.

Of course, this method will require some computational costs.

Also, the matrices of paired comparisons can be used for determining the evaluation of criterion conformity.

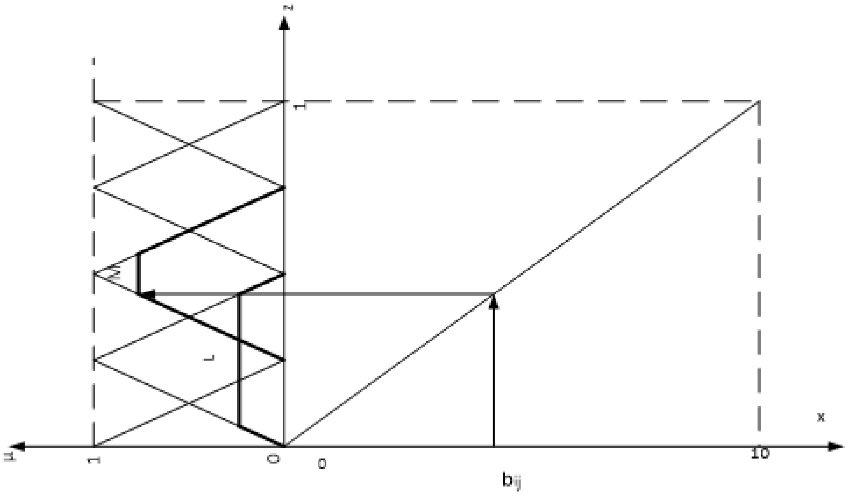
However, the amount of calculations is rapidly growing both with increasing number of criteria and increasing number of alternatives.

Moreover, when a large number of alternatives it is difficult to ensure the necessary level of coherence.

More simple is the method of score evaluations. At the same time, owing to their fuzzy character it is expedient to transform them into verbal form.

Suppose that to assess the degree of conformity of criteria to requirements was built a lot of linguistic assessments $L_p = \{l_q : q = \overline{1,Q})$ for example {*VL* - very low, *L* - low, *M* - medium, *H* - high, *VH* - very high} and corresponding fuzzy sets with membership functions $M = \{\mu_q(z) : z \in [0,1]\}$.

Figure 1 shows algorithm of transition from numerical score evaluations to verbal. Triangular membership functions are selected only for reasons of images simplicity.



Source: created by the authors.

Figure 1. **The Transition from Score Evaluations to Verbal**

As follows from the figure to some score b_{ij} of *i*-th alternative for *j*-th criterion correspond two linguistic evaluations: "small" and "medium", with the corresponding membership functions $\mu_L(b_{ij}),\mu_M(b_{ij})$, wherein $\mu_M(b_{ij})>\mu_L(b_{ij})$.

Membership functions can be interpreted as the distribution of the truths (possibilities) for the relevant evaluations.

Then, from *Figure 1* it follows that the truth of the linguistic evaluation "average" is larger than the truth of evaluation "small", and evaluation "average" can be taken as definitive.

Using only the value of the membership function may lead to a situation when several alternatives to will have the same assessments for all criteria at various values of the linguistic criterion of conformity. For elimination of this situation, we will use an integrated assessment:

$$\gamma= \mu_M\,(b_{ij})*CG(\mu_M(b_{ij})), \qquad (6)$$

where $CG(\mu_M(b_{ij}))$ – coordinate of centre gravity of the figure, bounded by the curve of the corresponding membership function. In the example shown in *Figure 1* in the transition to linguistic scores, they turn out in the form of fuzzy sets with trapezoidal membership functions.

In this case when calculating using equation (6) instead the centre of gravity coordinates can use the Chui and Park estimate (Chui, Chan, 1994):

$$Cp=(a1+a2+a3+a4)/4+w(a2+a3)/2,$$

where a1, a4, a2, a3 - the coordinates of the upper and lower bases of the trapezoidal membership function.

For symmetric trapeziums parameter, *w* can be put equal to unity. Suppose for definiteness that the combined agreed criteria system contains ten criteria, and the set of alternative variants contains four projects P_1, P_2, P_3, P_4 for which the results of the described transformations are presented in *Table 5*.

Table 5. Evaluation of projects at a combined system of criteria

Project / Criterion	Weight (w)	$\gamma(P_1)$	Weight (w)	$\gamma(P_2)$	Weight (w)	$\gamma(P_3)$	Weight (w)	$\gamma(P_4)$
C_1	0.1	[H]= 0.6	0.12	[M]=.45	0.14	[H] =.8	0.1	[VH]=0.8
C_2	0.1	[M]= 0.5	0.13	[H]=0.75	0.15	[VL]= 0.3	0.1	[L]= 0.3
C_3	0.15	[VH]= 0.8	0.2	[VH]= 0.85	0.13	[VH]= 0.8	0.2	[VH]= 0.95
C_4	0.08	[M]= 0.65	0.04	[M]=0.5	0.09	[L]= 0.35	0.09	[M]= 0.6
C_5	0.08	[H]= 0.7	0.04	[H]=0.8	0.06	[H]= 0.8	0.1	[H]= 0.75
C_6	0.12	[M]= 0.65	0.15	[H]=0.8	0.11	[M]= 0.6	0.08	[L]= 0.4
C_7	0.1	[L]= 0.4	0.11	[L]=0.4	0.04	[VH]= 0.85	0.06	[H]= 0.8
C_8	0.06	[L]= 0.25	0.09	[VH]=0.9	0.12	[M]= 0.5	0.08	[L]= 0.45
C_9	0.11	[M]= 0.6	0.07	[VH]=0.8	0.07	[H]= 0.7	0.12	[H]= 0.75
C_{10}	0.1	[H]= 0.7	0.05	[L]=0.35	0.09	[VL]= 0.25	0.07	[VL]= 0.3

Source: own calculations.

4. Rules for the Choice of Alternatives

The alternative choice rule can be written as the intersection (pessimistic position) or association (the optimistic position) of the corresponding fuzzy sets, which formalized by the operations of a minimum or maximum, respectively, executed for corresponding membership functions.

Then: 1) for equivalent criteria:

$$P^{pes} = \bigcap_{j=1}^{M} \gamma_{C_j}(P_i) = \min_{j=1,M} \gamma_{C_j}(P_i); i=\overline{1,I}. \quad P^{opt} = \bigcup_{j=1}^{M} \gamma_{C_j}(P_i) = \max_{j=1,M} \gamma_{C_j}(P_i); i=\overline{1,I};$$

2) for non-equivalent criteria:

$$P^{pes} = \bigcap_{j=1}^{M} \gamma_{C_j}^{\alpha_j}(P_i) = \min_{j=1,M} \gamma_{C_j}^{\alpha_j}(P_i) ; i=\overline{1,I}, \quad P^{opt} = \bigcup_{j=1}^{M} \gamma_{C_j}^{\alpha_j}(P_i) = \max_{j=1,M} \gamma_{C_j}^{\alpha_j}(P_i) ; i=\overline{1,I}, \quad (7)$$

where - α_j the coefficient importance of criterion C_j, $\sum_{j=1}^{M} \alpha_j = 1$.

The level of inconsistencies of project evaluations on all criteria will be equal:

$$\overline{P^{opt}} = 1 - P^{opt} = 1 - \mu_{C_j}(P_i).$$

The uncertainty of obtained estimates can be determined in the following way:

$$H = P^{opt} \wedge \overline{P^{opt}} = \mu_{C_j}(P_i) \wedge \overline{\mu_{C_j}(P_i)} = \min\{\mu_{C_j}(P_i), \overline{\mu_{C_j}(P_i)}\}.$$

When comparing the fuzzy sets $H, P^{opt}, \overline{P^{opt}}$, i.e. for the ordering of projects and determining the best alternative, is necessary to select those projects for which $P^{opt} \xrightarrow{\mu} 1$, H and $\overline{P^{opt}} \xrightarrow{\mu} 0$.

When no equilibrium criteria, using the data in *Table 5* and the relation (7), we obtain:

$P_1^{pes} = 0.91, P_1^{opt} = 0.97, P_2^{pes} = 0.9, P_2^{opt} = 0.99, P_3^{pes} = 0.83, P_3^{opt} = 0.98, P_4^{pes} = 0.89, P_4^{opt} = 0.99.$

The structuring of the set of projects with the pessimistic position has the form $P_3 > P_2 > P_1 = P_4$, with optimistic $P_1 > P_3 > P_2 = P_4$.

The inequality sign (>) is used to indicate the preference, the equal sign (=) for equivalence.

Is obvious that for "optimist" and "pessimist" there is no unequivocal choice, at least, between projects P_1, P_2, P_4, although P_4 project can be considered as the least interesting.

It should be noted the problem of influence of criteria weighting coefficients. The fact is that for the case of equivalent criteria it turns out is quite another structuring of projects: pessimistic – $P_3 > P_4 > P_2 > P_1$, optimistic – $P_1 > P_3 > P_2 > P_4$. Therefore, the appointment of the weights of criteria must be approached very carefully.

5. Consideration the Core of Criteria, Results of Calculations and Analyse

The above arguments have been done without taking into account content aspect of the criteria. Criteria space can be divided into two disjoint subspaces: elements of the first are criteria characterizing revenue opportunities, improving some indicators of development, etc., the second - the criteria that represent the possible expenses, any possible deterioration in the external environment of the project, associated with its implementation.

Let us denote these subspaces by $C^{(+)}$ and $C^{(-)}$ respectively. It is obvious that the convolution of estimates for the relevant criteria should be done in different ways.

For first (favourable) variant, in which case the estimates for the criteria belonging to $C^{(-)}$ must be minimal, i.e. $\min_j (P_i^{C_j^{(-)}})$ and the best will be $\min_i \min_j (P_i^{C_j^{(-)}})$, (8)

for the criteria belonging to $C^{(+)}$, the best will be variant $\max_i (\max_j P_j^{C_j^{(+)}})$. (9)

For second (adverse) variant for $C^{(-)}$ will be characterized by combination

$\min_i (\max_j P_i^{C_j^{(-)}})$ (10), for $C^{(+)}$ - $\max_i (\min_j P_j^{C_j^{(+)}})$ (11).

Suppose that the subspace $C^{(-)} = \{C_1, C_2\}$, respectively $C^{(+)} = \{C_3, C_4, ..., C_{10}\}$.

Results of calculations on the ratio (8)-(11) the case of no equilibrium criteria are shown in *Table 6*.

Table 6. Evaluation of projects on criteria C(-) and C(+)

Projects	Favorable variant		Adverse variant	
	min $P_i^{C_j^-}$	max $P_i^{C_j^+}$	max $P_i^{C_j^-}$	min $P_i^{C_j^+}$
P_1	0.93	0.97	0.95	0.91
P_2	0.91	0.99	0.96	0.90
P_3	0.83	0.98	0.97	0.88
P_4	0.89	0.99	0.98	0.91

Source: own calculations.

The obtained results again not allow obtaining an unambiguous decision.

The values of $\min P_i^{C_j^-}, \max P_i^{C_j^+}$, $\max P_i^{C_j^-}, \min P_i^{C_j^+}$ can be considered as material points belonging to some sets of project evaluations in appropriate situations (favourable or adverse). On these sets is theoretically possible to specify points $C_0^{(+)}$ or $C_0^{(-)}$, representing their integral estimates.

At this for the subspace $C^{(-)}$ point $C_0^{(-)}$ will characterize the maximum allowable "negative" assessment. The project will be the better, the farther its integral assessment for this set of criteria is located from the point $C_0^{(-)}$. In subspace, point $C_0^{(+)}$ characterizes the best for the particular situation integral evaluation of projects. The shorter the distance to this point of the integral evaluation for this group of criteria, the project is better.

The ranking of projects can be carried on the distance between the of projects received estimates and points $C_0^{(+)}$ or $C_0^{(-)}$ by distance $d^{(-)}(C_0^{(-)}, \min P_i^{C_j^-})$ or $d^{(+)}(C_0^{(+)}, \max P_i^{C_j^+})$ for a favorable variant or $d^{(-)}(C_0^{(-)}, \max P_i^{C_j^-})$, for adverse variant.

For criteria of groups C_j^- this distance should be the maximum, and for C_j^+ - minimal. Genuine values $C_0^{(+)}$ and $C_0^{(-)}$ practically impossible to determine. It is known [7], that the generalized characteristic of the system of material points is the coordinate of the centre of gravity, which in this case can be taken as a reference point.

If it is assumed that the mass of elements of the set is equal to unity, then calculating the coordinates of the centre of gravity can be replaced by calculating the average value

$$CG = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{for } m_i=1.$$

Distance to the ideal point can be calculated in different ways: Hamming, Euclidean distance, can be used the notion of pseudo inertia (Diday, 1979). In *Table 7* shows the values of linear distance.

Table 7. Evaluation of projects by the distance to the ideal point

Projects	Favorable case		Adverse case	
	$d^{(-)}(C_0^{(-)}, \min P_i^{C_j^-})$	$d^{(+)}(C_0^{(+)}, \max P_i^{C_j^+})$	$d^{(-)}(C_0^{(-)}, \max P_i^{C_j^-})$	$d^{(+)}(C_0^{(+)}, \min P_i^{C_j^+})$
P_1	0.04	0.0125	0.015	0.01
P_2	0.02	0.0075	0.005	0.0
P_3	0.06	0.0025	0.005	0.002
P_4	0	0.0075	0.015	0.01

Source: own calculations.

Unfortunately, the solution again turns ambiguous. Further attempts to obtain a unique solution are as follows: based on a pessimistic position for assessing the situation $C_j^{(-)}$ define

$$q_j = \min[d(C_j^{(-)}(favorable\_position), d(C_j^{(-)}(non\_favorable\_position)], \quad (12)$$

which gives the worst assessment of the negative factors for j-th project and

$$\xi_j = \max[d(C_j^{(+)}(favorable\_position), d(C_j^{(+)}(non\_favorable\_position)] \quad (13)$$

- the best estimate of favorable factors, and, finally, $\lambda_j = \frac{q_j}{\xi_j}$. (14)

The logic of this proposal is explained in that for a more attractive project $q_j \to \max$, $\xi_j \to \min$, which leads to increased evaluation λ_j.

Estimation (14) is relative and, as is known, it is more sensitive to the differences between the alternatives. Results of calculations on relations (12-14) are presented in *Table 8* and allow to make a final decision on the structuring of alternatives $P_1>P_2>P_3>P_4$.

Table 8. Evaluation of projects by relative index

projects	q_j	s_j	λ_j
P_1	0.015	0.0125	1.2
P_2	0.005	0.0075	0.67
P_3	0.005	0.002	0.25
P_4	0	0.01	0

Source: own calculations.

Conclusions

Thus, the paper proposes a method of structuring alternative solutions in the conditions of the uncertainty of expert estimates, which can be represented as fuzzy numbers or linguistic statements. For the situation when several experts or expert groups participate in the evaluation of alternatives with their own systems of criteria, a method is proposed to build a unified system of criteria based on the assessment of the truth of linguistic assumptions about the feasibility of including criteria in a unified system.

The principal feature of the proposed method of structuring alternatives is that the best alternative is chosen on the basis of an integrated assessment of the criteria characterizing the opportunities and threats that may occur in the implementation of the selected solution. The proposed method can be used in various applications where it is necessary to solve the problem of multi-criteria alternative choice in conditions of non-statistical uncertainty.

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DAUGIAKRITERINIS ALTERNATYVŲ PASIRINKIMAS PAGAL FUZZI INFORMACIJĄ

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SANTRAUKA

Daugiakriterinio alternatyvaus pasirinkimo problema, kad atliekant ekspertinį vertinimą, išlieka neapibrėžtumas ar laikomasi alternatyvių kriterijų reikalavimų. Apsvarstoma situacija, kai alternatyvų reitinge yra keli ekspertai, sudarantys individualią alternatyvų įvertinimo kriterijų sistemą, naudojant skaitines ar lingvistines formas. Straipsnyje siūloma sukurti vieningą kriterijų sistemą, įvertinant kalbinių teiginių apie kriterijų įtraukimo į vieningą sistemą galimybę, o vėliau nustatomi kriterijai, kurie apibūdina galimybes ir grėsmes, nesutampančių dalių įgyvendinant alternatyvas. Priešingai nei anksčiau atliktuose tyrimuose, šiame straipsnyje galutinis geriausios alternatyvos pasirinkimas yra grindžiamas integruotų įverčių santykiu pagal kriterijus, apibūdinančius galimybes ir grėsmes.

REIKŠMINIAI ŽODŽIAI: FUZZI rinkiniai, daugiakriterinis pasirinkimas, kalbinės vertės, vertinimo kriterijai.