The integrable nonlocal nonlinear Schrödinger equation: Riemann-Hilbert approach and long-time asymptotics

Dmitry Shepelsky, *Kharkiv, Ukraine*
Yan Rybalko, *Kharkiv, Ukraine*

We study the initial value problem for the integrable nonlocal nonlinear Schrödinger (NNLS) equation

$$iq_t(x,t) + q_{xx}(x,t) + 2q^2(x,t)\bar{q}(-x,t) = 0$$

with decaying (as $x \to \pm\infty$) boundary conditions as well as with the step-like boundary conditions: $q(x,0) \to 0$ as $x \to -\infty$ and $q(x,0) \to A$ as $x \to -\infty$, where $A \neq 0$.

The main aim is to describe the long-time ($t \to +\infty$) behavior of the solution of these problems. To do this, we adapt the nonlinear steepest-decent method to the study of the Riemann-Hilbert problem associated with the NNLS equation. In the case of decaying initial data, our main result is that, in contrast to the local NLS equation, where the main asymptotic term (in the solitonless case) decays to 0 as $O(t^{-1/2})$ along any ray $x/t = const$, the power decay rate in the case of the NNLS depends, in general, on x/t, and can be expressed in terms of the spectral functions associated with the initial data.

In the case of the step-like boundary conditions, the asymptotics turns to be different in different sectors of the (x,t) plane. Particularly, in the right-most sector, the main asymptotic terms is a constant depending on the ratio x/t whereas the second term decays, as in the previous case, with the power decay rate depending on x/t.

Vanishing of solution of the model representative of NPE

Kateryna Stiepanova, *Kharkiv, Ukraine*

The theory of quasilinear parabolic equations has been developed since the 50-s of the 19th century. The properties of these equations differ greatly from those of linear equations. These differences were revealed in the scientific papers of the mathematicians: Barenblatt G.I., Oleinic O.A., Kalashnikov A.S., Zhou Yu Lin and others. Specific properties of NE (inertia, strongweaked localization of solutions' supports, extinction...) were studied by J.I. Diaz, L. Veron, A.E. Shishkov, B. Helffer, Y. Belaud, D. Andreucci and others. The most important aspect of such investigations is the description of structural conditions affecting the appearance and disappearance of various non-linear phenomena. Our investigation deals with nonlinear parabolic equation with degenerating absorption potential h(t), the presence of which play the important role in the study of the above mentioned properties.

So, we study Cauchy-Neumann problem for the next type of a quasilinear parabolic equation with the model representative:

$$u_t - \Delta u + h(t)|u|^{q-1}\, u = 0 \quad \text{in}\, \Omega \times (0,T) \tag{1}$$

$$\frac{\partial u}{\partial n}|_{\partial\Omega\times[0,T]} = 0 \tag{2}$$

$$u(x,0) = u_0(x), \qquad \mathbb{R}^N \backslash \{\operatorname{supp} u_0\} \neq \emptyset, \{\operatorname{supp} u_0\} \subset \{|x| < 1\} \tag{3}$$

Here $0 < q < 1$, the initial function $u_0(x) \in L_2(\Omega)$, $\Omega \subset \mathbb{R}^N (N \geqslant 1)$ be a bounded domain with C^1 - boundary. Assume that h(t) is a continuous, nonnegative, nondecreasing function, such that $h(0) = 0$. Let $h(t) = exp(-\frac{\omega(t)}{t})$, where $\omega(t)$ satisfies following technical conditions: (A) $\omega(t) > 0 \;\; \forall\, t > 0$, (B) $\omega(0) = 0$, (C) $\frac{t\,\omega'(t)}{\omega(t)} \leq 1 - \delta \;\; \forall\, t \in (0, t_0)$, $t_0 > 0$, $0 < \delta < 1$.

Theorem 1. *Let be an arbitrary function from $L_2(\Omega)$, $\omega(t)$ is continuous and nondecreasing function satisfy assumptions (A)(B)(C), then an arbitrary solution $u(x,t)$ of the problem (1)(2)(3) vanishes on Ω in some finite time $T < \infty$.*

To prove that, we use local energy method, which deals with norms of solutions $u(x,t)$ only and, therefore, may applied for higher order equations too.
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One optimal control problem for an unmanned aerial vehicle

Yuliia Sukhinina, *Kharkiv, Ukraine*
Svetlana Ignatovich, *Kharkiv, Ukraine*

The papers [1, 2] deal with one problem of minimizing the time for a kinematic model of unmanned aerial vehicle moving at a constant altitude. From a kinematic point of view, an UAV flying at a constant altitude is determined by standard Dubins equations [3]. Under additional speed constraints, the flight model of a drone is described by the following system of differential equations:

$$\dot{x} = \cos\theta, \quad \dot{y} = \sin\theta, \quad \dot{\theta} = u, \tag{1}$$