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The Static and Dynamic Hysteresis Responsible for the Dislocational Amplitude Dependent Internal Friction

I. Theory

By

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A dynamic theory of the amplitude dependent dislocation hysteresis is developed, emphasising the importance of the quasi-particle–dislocation interaction for the non-linear response of the real crystal to the high-frequency ultrasonic wave propagation. A strong frequency dependence of the sound attenuation rate is predicted, with a characteristic maximum near the transition from the dynamic to the static hysteresis. In the latter case the transition to the limit gives the basic results of the Granato-Lücke and Rogers theories describing the amplitude dependent internal friction due to underdamped dislocation loops. The dynamic theory of the dislocation hysteresis provides an explanation to the anomaly shown by the high-frequency, amplitude dependent internal friction of metals at the N–S transition. As is seen from the analysis, it is directly related to the decrease in the S-state of the electron drag to dislocation.

Eine dynamische Theorie der amplitudenabhängigen Versetzungshysteresis wird entwickelt und auf die Bedeutung der Quasiteilchen–Versetzungs-Wechselwirkung für die nichtlineare Response des Realkristalls auf eine hochfrequente Ultraschallausbreitung hingewiesen. Eine strenge Frequenzabhängigkeit der Schalldämpfungsrate wird vorausgesagt mit einem charakteristischen Maximum in der Nähe des Übergangs von der dynamischen zur statischen Hysteresis. Im letzteren Falle ergibt der Grenzübergang die grundlegenden Ergebnisse der Granato-Lücke- und Rogers-Theorien, die die amplitudenabhängige innere Reibung infolge von unterdämpften Versetzungsschleifen beschreiben. Die dynamische Theorie der Versetzungshysteresis liefert eine Erklärung der Anomalie, die sich in der hochfrequenten, amplitudenabhängigen inneren Reibung der Metalle zum N–S-Übergang zeigt. Aus der Analyse ist ersichtlich, daß sie direkt mit dem Abfall des S-Zustands des Elektronendrags zu Versetzungen verbunden ist.

1. Introduction

Many properties of real crystal are determined by the interaction of dislocations with point defects. In particular, it is responsible for the hysteresis nature of the stress–strain relation resulting in an amplitude dependence of internal friction. The phenomenon was analysed by Granato and Lücke [1]. Rogers [2] extended the theory to high stress amplitudes and analysed the amplitude dependence of dynamical losses. Later on, several attempts were made of including into the analysis the thermally activated nature of obstacle surmounting by dislocations [3 to 7]. However, one of the most significant aspects of the dislocation–pinning centre interaction under alternating stress was not considered. The point is that the theory [1 at 7] was developed for the underdamped motion of dislocation. In the case of an overdamped motion the physical situation would be far more complex, e.g., the condition determining unpinning of a dislocation segment pair from their common pinning centre

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would depend upon viscosity. This would result in a different value of the unpinning stress [8, 9] and also influence the number of loops to be unpinned at a given external stress level. Hence, the instantaneous distribution function of loop lengths should be dependent on the amount of damping. Besides, the presence of viscosity might give rise to such a performance that unpinning of dislocations from their pinning centre would not be "catastrophic". In other words, unpinning of a dislocation from one centre would not provoke spreading of the break-away along the entire loop length L_N.

However, the main role of viscosity in the amplitude dependent region consists of its direct effect upon the so-called hysteresis losses. In the ultimate account the latter are also due to interaction between the elastic fields of mobile dislocations and various quasi-particles in the crystal. Prior analyses of these losses all ignored the fact that the quasi-particles interact with dislocations, hence produce a dynamic drag force [9 to 11] and exert influence on the dislocation hysteresis. With an increase in the vibration frequency the amount of that influence can grow to such an extent as to alter the very nature of the hysteresis. As will be shown below, the amplitude dependent losses are mainly controlled, in the case of overdamped dislocation motion, by the dissipation of energy during viscous motion of the dislocation loops L_N. They are inversely proportional to the forced vibration frequency ω and the dislocation drag constant B. There are possibilities for explaining quite a number of other effects manifesting themselves in the amplitude dependent region at high frequencies as a result of the strong influence of viscosity upon the dynamics of dislocations. This allows, in particular, the increase in the dislocation amplitude dependent internal friction of metals observed at the N–S transition [12, 13] to be related to a decrease in electron viscosity.

2. Effect of Viscosity on Forced Vibrations of a Pinned Dislocation

Analysis of forced vibrations of a pinned dislocation in a viscous environment is one of the necessary elements of a consistent dynamic theory of the dislocations amplitude dependent hysteresis. The problem can be most simply formulated in the framework of the string model where the equation of dislocation motion under the action of an external stress $\sigma_0 \sin \omega t$ has the form

$$M \frac{\partial^2}{\partial t^2} U(x, t) + B \frac{\partial}{\partial t} U(x, t) - C \frac{\partial^2}{\partial x^2} U(x, t) = b\sigma_0 \sin (\omega, t) . \tag{1}$$

Here M and C denote the linear density of the dislocation effective mass and the linear tension constant, respectively, $U(x, t)$ is the displacement from the equilibrium position, and b the value of the Burgers vector. The solution of (1) representing forced vibrations of a dislocation segment of length L fixed at the ends $x_1 = -L/2$ and $x_2 = L/2$ to point defects can be written in closed form, viz.

$$U(x, t) = \frac{b\sigma_0}{C} \operatorname{Im} \left[e^{i\omega t} \frac{\cos \varkappa x - \cos \varkappa L/2}{\varkappa^2 \cos \varkappa L/2} \right], \tag{2}$$

where $\varkappa^2 = (M/C)\, \omega(\omega - i\gamma)$ and $\gamma = B/M$. Starting from this equation we will analyse the effect of viscosity on the angle $\varphi(t)$ formed by the tangent to the dislocation line at one of the pinning points and the straight line connecting these parts. Taking the derivative of $U(x, t)$ with respect to x at $x = -L/2$ we find

$$\tan \varphi(t) = \frac{b\sigma_0}{C} \operatorname{Im} \left[e^{i\omega t} \frac{\tan \varkappa L/2}{\varkappa} \right]. \tag{3}$$

Despite the apparent simplicity of this formula it leads to a rather cumbersome equation for the "angle of attack" tangent, $\varphi(t)$ (cf. [14]). Therefore, it seems reasonable to make use of the smallness of ω compared with γ. Note that because of the electron viscosity alone, γ of normal metals, γ_n, reaches values of the order 10^{10} to 10^{11} s^{-1} [9]. In the superconducting state γ $(= \gamma_s)$ decreases sharply at lower T following the formula [9]

$$\gamma_s = \gamma_n \frac{2}{1 + \exp\left[\Delta(T)/T\right]}, \tag{4}$$

where $\Delta(T)$ is the energy gap of the superconductor. However, even at rather low T, e.g. $T = 0.25T_c$ (T_c being the critical temperature of the superconductor) the values assumed by γ_s are still high as compared with ω generally employed ($\approx 10^7$ to 10^8 s^{-1}). In addition, it should be borne in mind that (4) is only valid for low velocities of the dislocation elements, viz. $V \ll T/\hbar q_m$; $\Delta(T)/\hbar q_m$ where q_m is the characteristic size of a Brillouin cell [9]. It cannot be excluded, however, that in more general cases these velocities can reach or even exceed, during some parts of the vibration period, the critical value $V_c = 2\Delta(T)/\hbar q_m$ marking the break of Cooper pairs. Since this leads to an increase of the drag force, the situation is even more favourable for the inequality $\omega \ll \gamma$ to hold.[2]) We will consider only small values of φ, otherwise the dislocation pinning condition cannot be given any physical meaning with typical values of the dislocation–point defect bonding force. Using (3) we can write

$$\varphi(t) = \varphi_0 \sin(\omega t - \Phi)\,, \tag{5}$$

where the amplitude of the "angle of attack" and its phase shift with respect to the external stress are given by

$$\left.\begin{aligned} \varphi_0 &= \varphi_d \left[\frac{\cosh\dfrac{\sqrt{2}\,L}{L_d} - \cos\dfrac{\sqrt{2}\,L}{L_d}}{\cosh\dfrac{\sqrt{2}\,L}{L_d} + \cos\dfrac{\sqrt{2}\,L}{L_d}}\right]^{1/2} \\ &\text{and} \\ \Phi &= \arctan\left[\frac{\sinh\dfrac{\sqrt{2}\,L}{L_d} - \sin\dfrac{\sqrt{2}\,L}{L_d}}{\sinh\dfrac{\sqrt{2}\,L}{L_d} + \sin\dfrac{\sqrt{2}\,L}{L_d}}\right] \end{aligned}\right\} \tag{6}$$

with $0 \leqq \Phi \leqq \pi/4$. Here $\varphi_d = b\sigma_0/\sqrt{CB\omega}$ is the magnitude assumed by the amplitude φ_0 when the dislocation segment forced vibrations are overdamped; $L_d = 2\sqrt{C/B\omega}$ is the characteristic damping length, an important parameter of the theory developed. For further purposes it will be rather useful to introduce the effective dislocation segment length $\mathcal{L}$ related with $\varphi_0 = b\sigma_0\mathcal{L}/2C$ by the same equation as in the absence of viscosity.

In Fig. 1a and b the φ_0/φ_d vs. L/L_d and Φ vs. L^2/L_d^2 dependences are plotted. Since $\mathcal{L}/L_d = \varphi_0/\varphi_d$ by definition, the $\mathcal{L}(L)$ dependence can be well approximated by the simple function

$$\mathcal{L}(L) = \begin{cases} L & \text{at} \quad L < L_d\,, \\ L_d & \text{at} \quad L \geqq L_d\,. \end{cases} \tag{7}$$

[2]) Note that in the region of parameter values where the mechanism of Cooper pair break is active, the constant γ can be defined only in some effective sence, since the velocity dependence of the drag force is essentially non-linear here [9].

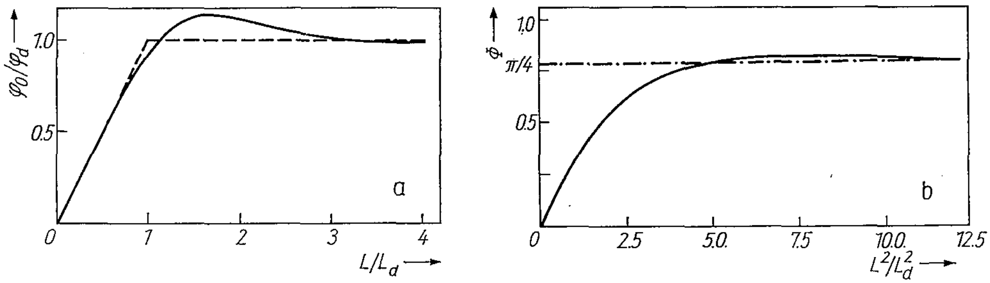

Fig. 1. a) The effect of the amount of damping of a vibrating dislocation segment upon the normalized amplitude of its "angle of attack" and b) the phase shift with respect to the applied stress

This can be easily seen in Fig. 1a. The approximate representation on $\mathcal{L}(L)$ greatly facilitates the account of the influence of viscosity upon unpinning from their pinning centres.

Another important dynamic characteristic of a dislocation segment performing forced vibrations is its mean displacement from the equilibrium position,

$$\overline{U}(t) = \frac{1}{L} \int\limits_{-L/2}^{L/2} U(x, t)\, dx\,.$$

Substituting (2) in the integrand and carrying out the integration, we obtain

$$\overline{U}(t) = \frac{b\sigma_0}{C} \operatorname{Im} \left[e^{i\omega t} \frac{\frac{2}{\varkappa L} \tan \frac{\varkappa L}{2} - 1}{\varkappa^2} \right].$$

Bringing this to the form

$$\overline{U}(t) = \overline{U}_0 \sin(\omega t - \psi)\,, \tag{8}$$

we can find the mean displacement amplitude of the segments and its phase shift from the external stress. With $\omega \ll \gamma$ the two values are given by

$$\left.\begin{aligned} \overline{U}_0 &= \overline{U}_d \left[1 - 2\frac{\mathcal{L}}{L} \cos \Phi + \frac{\mathcal{L}^2}{L^2}\right]^{1/2} \\ &\text{and} \\ \psi &= \arctan \left[\frac{L - \mathcal{L} \cos \Phi}{\mathcal{L} \sin \Phi}\right], \end{aligned}\right\} \tag{9}$$

respectively, with $0 \leqq \psi \leqq \pi/2$; here $\overline{U}_d = b\sigma_0/B\omega$ is the displacement amplitude of an overdamped segment, $\mathcal{L}$ and Φ are assumed to be expressed in terms of L with the aid of (6) and the known relation of $\mathcal{L}$ to φ_0. Fig. 2a and b represent the $\overline{U}_0/\overline{U}_d$ vs. L^2/L_d^2 and ψ vs. L^2/L_d^2 dependences. Similar as in the foregoing figures, the curves almost merge with their asymptotic representations as soon as L becomes a few times larger than L_d.

These results suggest that the vibrating dislocation segment is overdamped at $L \gtrapprox L_d$. The mode of its motion is virtually the same as that of an unpinned dislocation moving as a whole under the action of a sinusoidal stress (save the sections directly adjacent to the pinning points). There is a rather wide range of segment lengths, $L \lessapprox L_d$, where viscosity exerts a substantial influence on the segment dynanic. The effect of viscosity can be neglected only at $L \ll L_d$.

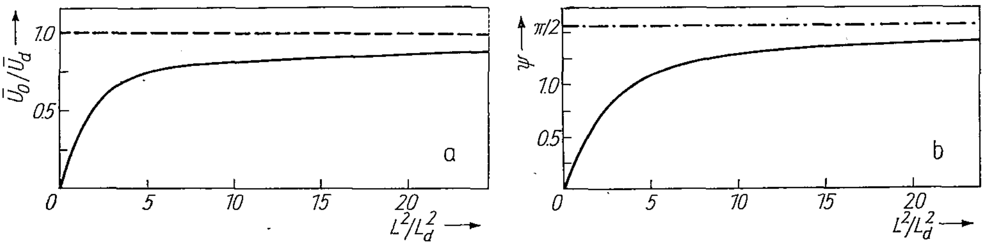

Fig. 2. a) The effect of the amount of damping of a vibrating dislocation segment upon the normalized amplitude of its mean displacement and b) the phase shift with respect to the applied stress

3. Effect of Viscosity on Dislocation Unpinning from Their Pinning Centres

As a factor responsible for the dynamics of vibrating dislocation segments, viscosity has a direct effect on the process of dislocation unpinning from their pinning centres under the action of an applied stress. To analyse this effect, first consider the force acting on an arbitrary pinning centre on the part of adjacent dislocation segments of lengths L_1 and L_2, respectively. Using the linear dislocation tension approximation, and recalling (5) and the definition of $\mathscr{L}$, the force can be written as

$$f(t) = f_0 \sin(\omega t - \theta)\,,$$

where the force amplitude f_0 and the phase shift θ with respect to the applied stress are

$$\left.\begin{aligned} f_0 &= \frac{b\sigma_0}{2}\,[\mathscr{L}_1^2 + 2\mathscr{L}_1\mathscr{L}_2\cos(\Phi_1 - \Phi_2) + \mathscr{L}_2^2]^{1/2} \approx \frac{b\sigma_0}{2}\,(\mathscr{L}_1 + \mathscr{L}_2) \\ \text{and} \quad \theta &= \arctan\left[\frac{\mathscr{L}_1 \sin \Phi_1 + \mathscr{L}_2 \sin \Phi_2}{\mathscr{L}_1 \cos \Phi_1 + \mathscr{L}_2 \cos \Phi_2}\right] \end{aligned}\right\} \tag{10}$$

with $0 \leqq \theta \leqq \pi/4$. The approximate representation for f_0 yields the highest errors at $L_1 \lessapprox L_\mathrm{d}$ and $L_2 \gtrapprox L_\mathrm{d}$, or $L_1 \gtrapprox L_\mathrm{d}$ and $L_2 \lessapprox L_\mathrm{d}$. However, as can be seen in the plots of Fig. 1a and b, it is less than 10% even in these cases. With $L_1, L_2 \ll L_\mathrm{d}$ or $L_1, L_2 \gg L_\mathrm{d}$ the error is quite insignificant. Hence, the effective segment lengths are practically additive.

Now we take into account that the dislocation segments cannot get unpinned from their common pinning centre unless f_0 exceeds the maximum dislocation-centre bonding force $f_{\max}$. This condition can be written as

$$\mathscr{L}(L_1) + \mathscr{L}(L_2) > \frac{2f_{\max}}{b\sigma_0} = \mathscr{L}_{\sigma_0}\,. \tag{11}$$

In contrast to the unpinning condition used in [1], this equation involves effective segment lengths, hence takes into account the viscosity.[3] Generally, this should bring about a dependence of the critical stress amplitude σ_c upon viscosity. Since the first to be unpinned are the longest segments among those present in the crystal, the unpinning criterion given above implies

$$\sigma_\mathrm{c} = \frac{f_{\max}}{b\mathscr{L}(L_{\max})}\,.$$

[3]) To avoid misunderstanding, we would like to emphasize that the region of validity of (11) is limited to forced vibrations of dislocation segments.

Here L_{max} is the upper limit to the dislocation segment lengths. In view of the random distribution of defects along the dislocation, L_{max} can be assumed approximately the same as L_N. Note that the existence of σ_c is a direct sequence of the existence of the upper limit $\mathcal{L}(L_{max})$ to the effective segment length. According to (7), the latter magnitude reaches its limiting value L_d at $L_{max} \approx L_N > L_d$ and we have for

$$\sigma_c = \frac{f_{max}}{bL_d} = \frac{f_{max}}{2b}\sqrt{\frac{\omega B}{C}}\,. \tag{12}$$

This differs from Mason's result [8] by the factor $\sqrt{\omega B/C}$, the discrepancy being due to the fact that Mason neglected the influence of viscosity upon the shape of the vibrating dislocation. The effect of viscosity on σ_c was also considered in [9] but the approximate formula for σ_c given in that paper looks unreasonably complex and seems to represent the qualitative aspect of the dependence only.

Another side of the problem considered here, i.e. the effect of viscosity on dislocation unpinning, is the role played by viscosity in controlling the unpinning probability of dislocation loops L_N pinned to point defects. Let us calculate the probability $P(\sigma_0)$ that two adjacent dislocation segments would not get unpinned from their common pinning centre at $\sigma_0 > \sigma_c$ during the entire period of vibrations. It is clear enough that the said probability is equal to that of finding the segment lengths in the region S_{σ_0} where the inequality of (11) does not hold. In other words, with L_1 and L_2 belonging to S_{σ_0} we would have[4])

$$\mathcal{L}(L_1) + \mathcal{L}(L_2) < \mathcal{L}_{\sigma_0} < 2\mathcal{L}(L_N)\,. \tag{13}$$

It should be noted at once that $L_N < L_d$ leads to $L_1, L_2 < L_d$. The latter inequality should also hold with $L_N < L_d$, provided that $L_{\sigma_0} < L_d$ (otherwise (13) would not be true). Hence in such cases $\mathcal{L}(L_1) = L_1$ and $\mathcal{L}(L_2) = L_2$. Taking this into account in (13), we come to the conclusion that with $L_N < L_d$ or $L_N < L_d$ and $\mathcal{L}_{\sigma_0} < L_d$ both the region S_{σ_0} and the probability P_{σ_0} are described by the same formulas as with no viscosity.[5]) Hence, here $P(\sigma_0)$ can be represented by the formula obtained by Granato and Lücke [1], viz.

$$P(\sigma_0) = 1 - \left(\frac{\Gamma}{\sigma_0} + 1\right)\exp\left(-\frac{\Gamma}{\sigma_0}\right),$$

where $\Gamma = 2f_{max}/bL_c$ denotes the characteristic stress level and L_c the mean segment length.

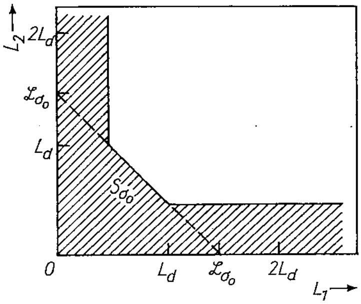

Fig. 3. The range of dislocation segment lengths not obeying the condition of unpinning from their common pinning centre under the action of a sinusoideal external stress

[4]) The inequality $\mathcal{L}_{\sigma_0} < 2\mathcal{L}(L_N)$, or more exactly $\mathcal{L}_{\sigma_0} < 2\mathcal{L}(L_{max})$ implies $\sigma_0 > \sigma_c$.

[5]) The statement concerning $P(\sigma_0)$ is true only if the effect of viscosity upon Kohler's distribution function of dislocation segment lengths can be neglected.

Let us now analyse the situation characterized by $L_N > L_d$ and $L_d < \mathcal{L}_{\sigma_0} < 2L_d$. Making use of the above-mentioned approximate representation of $\mathcal{L}(L)$, one can easily see that S_{σ_0} has the form as shown in Fig. 3. Since the probability wanted is defined by the integral

$$P(\sigma_0) = \iint\limits_{S_{\sigma_0}} \exp\left(-\frac{L_1 + L_2}{L_c}\right) \frac{dL_1\, dL_2}{L_c^2},$$

we have

$$P(\sigma_0) = 1 - \left(\frac{\Gamma}{\sigma_c} - \frac{\Gamma}{\sigma_0} + 1\right) \exp\left(-\frac{\Gamma}{\sigma_0}\right).$$

With σ_c expressed by (12), $P(\sigma_0)$ is obviously dependent on viscosity. Besides, in the range of stress amplitudes $\sigma_c < \sigma_0 < 2\sigma_c$ the role of viscosity manifests itself through the semicatastrophic mode of dislocation unpinning from their pinning centres. This follows directly from the fact that fulfilment of the unpinning condition (11) does not necessary imply the validity of the propagation condition $\mathcal{L}(L_1 + L_2) + \mathcal{L}(L_1) > \mathcal{L}_{\sigma_0}$. Note that all the considerations presented relate only to $L_N > L_d$ and $L_d < \mathcal{L}_{\sigma_0} < 2L_d$. With either $L_N < L_d$ or $L_N > L_d$ and $\mathcal{L}_{\sigma_0} < L_d$, unpinning of dislocation segments from their pinning centres is a catastrophic process, i.e. depinning of some segments necessarily involves unpinning of the adjacent ones.

The probability for none of the segment pairs placed along the dislocation line of length L_N to get unpinned over the entire vibration time is approximately equal to $P^n(\sigma_0)$ where $n = L_N/L_c - 1$ is the mean number of pairs. Hence, the probability of at least one pair to get unpinned can be expressed as $1 - P^n(\sigma_0)$. In case the unpinning process is of catastrophic nature, $1 - P^n(\sigma_0)$ coincides with the probability $P_N(\sigma_0)$ for the loop to get unpinned from all of its pinning points.[6] As a result, the total lengths of dislocation loops unpinned within a unit volume of crystal would be

$$L_N \frac{\Lambda}{L_N} \left\{1 - \left[1 - \left(\frac{\Gamma}{\sigma_0} + 1\right) \exp\left(-\frac{\Gamma}{\sigma_0}\right)\right]^n\right\}; \tag{14a}$$

Λ denoting the dislocation density. In the opposite case, i.e. when the dislocation depinning process does not spread to the entire length L_N, the total number of segments unpinned in a unit volume is given by

$$L_{\sigma_0} \frac{\Lambda}{L_N} \left\{1 - \left[1 - \left(\frac{\Gamma}{\sigma_c} - \frac{\Gamma}{\sigma_0} + 1\right) \exp\left(-\frac{\Gamma}{\sigma_0}\right)\right]^n\right\}, \tag{14b}$$

where L_{σ_0} is the mean value of the total unpinned length per loop length L_N. To calculate L_{σ_0} first note that in this case effective lengths of all segments formed as the result of unpinning events is the same, namely L_d. This is true even for unpinning from a single pinning centre. Indeed, as can be seen from Fig. 3, the inequality $L_1 + L_2 < L_d$ holds outside the region S_{σ_0}, hence $\mathcal{L}(L_1 + L_2) = L_d$. In other words, those of the segments would not be unpinned whose length L satisfies the condition $\mathcal{L}(L) < \mathcal{L}(\sigma_0) - L_d$. The latter can be simplified by replacing $\mathcal{L}(L)$ with L, as the right-hand side still proves smaller than L_d, due to the constraint $L_d < \mathcal{L}_{\sigma_0} < 2L_d$.

[6]) The value is different from the M of [1] in the respect that it represents the maximum unpinning probability, realizable over the vibration half-period, rather than the probability at a given moment. The necessity for such an approach will be evident later.

Finally, we can write for

$$L_{\sigma_0} = L_N - \frac{L_N}{L_c} \int\limits_0^{\mathcal{L}_{\sigma_0} - L_d} L \exp\left(-\frac{L}{L_c}\right) \frac{dL}{L_c} =$$

$$= L_N \left(\frac{\Gamma}{\sigma_0} - \frac{\Gamma}{2\sigma_c} + 1\right) \exp\left(\frac{\Gamma}{2\sigma_c} - \frac{\Gamma}{\sigma_0}\right).$$

Taking into account this result and (14a) and (14b), the total length of dislocation loops unpinned in a unit volume can be written in the general case as $\Lambda P_N(\sigma_0)$ where $P_N(\sigma_0)$ at early stages of the unpinning process (i.e. when σ_0 is small compared to Γ) is given by the expressions

$$P_N(\sigma_0) = \\ = \begin{cases} \frac{L_N}{L_c}\left(\frac{\Gamma}{\sigma_0} + 1\right) \exp\left(-\frac{\Gamma}{\sigma_0}\right) & \text{at either } L_N < L_d \text{ or } L_N > L_d \text{ and } \sigma_0 > 2\sigma_c\,, \\ \frac{L_N}{L_c}\left(\frac{\Gamma}{\sigma_0} - \frac{\Gamma}{2\sigma_c} + 1\right)\left(\frac{\Gamma}{\sigma_c} - \frac{\Gamma}{\sigma_0} + 1\right) \exp\left(\frac{\Gamma}{2\sigma_c} - \frac{2\Gamma}{\sigma_0}\right) & \text{at } L_N > L_d \text{ and } \sigma_c < \sigma_0 < 2\sigma_c\,. \end{cases} \tag{15}$$

Unlike the first formula, the second one of course cannot be treated as the probability for the loop L_N to get unpinned from all of its pinning points, as the situation simply does not exist with semicatastrophic unpinning. Assuming that the value (14b) is the result of total unpinning of some fictitious loops L_N one could interpret the corresponding expression for $P_N(\sigma_0)$ in the same sense as before. Then (15) would imply that viscous renormalization of the probability for a dislocation loop L_N to be unpinned from all of its pinning defects only occurs in the case of overdamped motion and within a finite range of applied stress amplitudes.

4. Effect of Viscosity on Dislocation Hysteresis

The quasi-static hysteresis analysed in [1] is determined by the dislocation drag due to point defects which is similar to dry friction. Meanwhile, because of interaction with various quasiparticles in the crystal, the dislocations experience viscous friction as well, whose significance increases with an increase of ω. In the high-frequency range it becomes the dominant effect forming the dislocation hysteresis loop. It seems noteworthy that the influence of viscosity is essentially non-linear, which manifests itself through the lack of additivity of the energy losses determined by dislocation unpinning from point defects, on the one hand, and their motion through the viscous medium, on the other. Analysis shows the additivity to take place only at low enough frequencies, $\omega \ll 4C/BL_N^2$ where the dynamic effects are too low in magnitude to have any impact on the quasi-static hysteresis. As the frequency increases, their influence becomes noticeable. At $\omega \gg 4C/BL_N^2$ the quasi-static hysteresis disappears, being replaced by a purely dynamic one. The way it occurs can be followed in Fig. 4a and b showing, in terms of reduced coordinates, the stress dependence of the mean displacement of the dislocation loop L_N moving in the underdamped and overdamped regime. The points B and B′ correspond to collisions of the loop L_N with point defects. Upon being pinned by the defects, the loop bends between them under the action of the growing applied stress and finally breaks away (points C and C′). Further on it performs a markedly non-stationary motion followed by the forced vibration regime (beginning at D and D′). According to (8), the loop L_N collides with point defects at

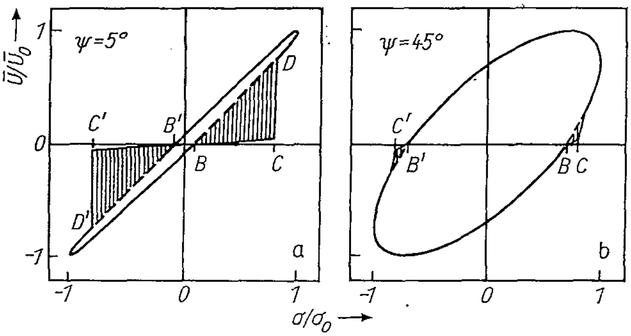

Fig. 4. The dislocation hysteresis in the a) underdamped and b) overdamped case. The area of the static hysteresis loop is hatched

external stresses of magnitude $|\sigma_0 \sin \varphi_N|$. If this value exceeds the level where the loop can get unpinned from all of its pinning defects, the hindering effect of these latter would be felt, but only very slightly, especially with short segments $(L \ll (2\pi/B)\sqrt{MC})$ whose unpinning is greatly favoured by the inertia effect [15]. Hence, one could say that the dislocation moves past the defects without taking notice of them. This situation corresponds to the case when the points C and C′ are inside the dynamic hysteresis loop, i.e. the static hysteresis is completely absent. Generally the dislocation hysteresis reveals both static and dynamic features.

Appealing directly to the amplitude dependent dislocational internal friction in the presence of viscosity, we first notice that the major part of the loops L_N getting unpinned over one alteration period of the applied stress corresponds to stresses σ close in absolute magnitude to σ_0, as a result of the exponential distribution in segment lengths.[7]) Making use of this fact, we can write, to a good accuracy, the following equation for energy losses in a unit crystal volume over the period $T = 2\pi/\omega$:

$$\Delta W = \Delta w_N(\sigma_0) \frac{\Lambda}{L_N} P_N(\sigma_0) , \tag{16}$$

where $\Delta w_N(\sigma_0)$ are energy losses of a single loop L_N being unpinned from point defects at an amplitude magnitude of the applied stress σ_0 and over the period T of the loop motion; $(\Lambda/L_N)\ P_N(\sigma_0)$ is the total number of loops getting unpinned in the unit volume of the crystal per complete cycle of alteration in $\sigma(t)$. The probability $P_N(\sigma)$ being known (see (15)), the problem reduces to calculating $\Delta w_N(\sigma_0)$ which is given by the integral

$$\Delta w_N(\sigma_0) = L_N b \oint \bar{U}_n(\sigma)\, d\sigma . \tag{17}$$

The integration is meant over the applied stress period, and $\bar{U}_N(\sigma)$ denotes the mean loop displacement which is regarded as a function of the instantaneous value of $\sigma(t)$.

To establish the functional dependence $\bar{U}_N(\sigma)$ generally is far from being a trivial problem, in view of the complex motion of the loop L_N during its break-away from the pinning centres and further build-up of the local vibration regime controlled by the applied stress. It is easily solvable in the quasi-static case analysed by Granato and Lücke [1], since at low frequencies the break-away can be considered to occur instantaneously (compared with a quarter of the vibration period), and the loop unpinned "adapts" very quickly to the applied stress (cf. [1]). The situation is even more simplified by the fact that the loop L_N, undergoing elastic contraction during the unload part of the stress alteration cycle, collides with point defects at very low stresses actually close to zero. Hence, there is no need for taking into account the

[7]) It should be underlined once again that we speak of early stages of the unpinning process.

inertia effect [15] which, as has been shown in [16], can manifest itself at $\sigma_0 \sin \psi_N$ greater than one half of the stress necessary for breaking the dislocation away from an obstacle by a static force. For unpinning the loop L_N at the amplitude stress σ_0 the condition is met at $\psi_N > \pi/6$ which is equivalent, as can be seen in Fig. 2b, to the demand that the loop be overdamped. Therefore, at $\omega > \omega_d$ ($\omega_d = 4C/BL_N^2$ being the characteristic damping frequency) the $\overline{U}_N(\sigma)$ dependence should be analysed with account of the inertia effect. In this connection we would like to point out the considerable importance which seems to have been overlooked by many writers. The concept of inertial surmounting of local obstacles by dislocations [15] has been analysed in a model based on the assumption that the dislocation segments into which mobile dislocations are divided by obstacles obey a δ-like length distribution. In this case the dislocations, getting unpinned from all the pinning centres at a time, naturally can surmount them at a twice lower stress than the critical break-away value owing to the additional bending of the segments. Whereas for distributions characterized by some reasonable amount of smearing, e.g. the Kohler distribution, the situation does not seems probable. Indeed, the break-away process is started through the inertial mechanism at one or a few favourable sites only and cannot spread through the entire loop length L_N because of the severe damping of the segment dislocations. In other words, the inertia effect results here in a much smaller reduction of the unpinning stress for the entire loop L_N, as compared with the δ-distribution.

With an increase in the stress $\sigma_0 \sin \psi_N$ at which the dislocation collides with point defects, the number of points increases where the break-away is initiated by the inertia mechanism. Hence, the importance of the inertia effect increases at higher amounts of loop overdamping.[8]) But at the same time the area occupied by the static hysteresis loop sharply decreases (cf. Fig. 4a and b), hence the contribution of the inertia effect to the total hysteresis losses can be neglected. The hysteresis loop at the dislocation considered, that gets unpinned from defects at the stress σ_g, would represent an ellipse with rectangular protuberances whose points C′ and C (Fig. 4) would have coordinates (−1, 0) and (1, 0), respectively. According to (8), the ellipse is given by the equation

$$\left(\frac{\overline{U}_N}{\overline{U}_{0N}}\right)^2 - 2\cos\psi_N \frac{\overline{U}_N}{\overline{U}_{0N}}\frac{\sigma}{\sigma_0} + \left(\frac{\sigma}{\sigma_0}\right)^2 = \sin^2\psi_N .$$

Calculating the integral of (17), i.e. the area bounded by this curve, and substituting the result into (16) we can find the ultrasound attenuation rate, viz.

$$\delta_H = \frac{\Delta W}{\sigma_0^2/G} = \frac{Gb^2}{4C} \Lambda L_d^2 \frac{\overline{U}_{0N}}{\overline{U}_{dN}} \left[\cos\psi_N + \left(\frac{\pi}{2} + \psi_N\right)\sin\psi_N\right] P_N(\sigma_0) , \tag{18}$$

where G is the shear modulus. With $L_N \ll L_d$ it is easy to obtain, making use of (6), (9), and (15), the result of Granato and Lücke [1] and Rogers [2], namely

$$\delta_H = \frac{Gb^2\Lambda}{12C}\frac{L_N^3}{l_c}\left(1 + \frac{\pi B\omega L_N^2}{20C}\right)\left(\frac{\Gamma}{\sigma_0} + 1\right)\exp\left(\frac{-\Gamma}{\sigma_0}\right).$$

With $L_N \gg L_d$ we have

$$\delta_H = \frac{\pi G b^2 \Lambda}{B\omega} P_N(\sigma_0) ,$$

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[8]) It is implied that B remains constant while ω and L_N change.

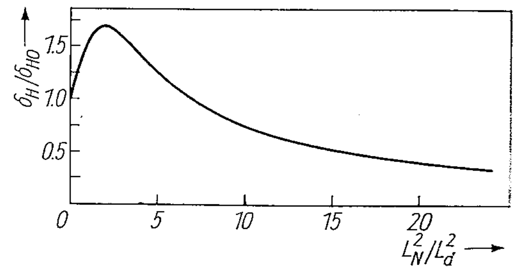

Fig. 5. The effect of the amount of dislocation loop damping upon the amplitude dependent absorption of ultrasonic waves in a crystal with a three-dimensional array of dislocations

where the unpinning probability $P_N(\sigma_0)$ is given by (15). As was seen, δ_H in the overdamped case happens to be inversely proportional to ω and B. Fig. 5 represents the dependence of (18), i.e. the ultrasound attenuation rate vs. the dislocation loop damping at $\sigma_0 > 2\sigma_c$. As is seen in the figure, the theory predicts a maximum of the attenuation at $\omega \approx \omega_d$.

Viscosity affects not only the ultrasound absorption but also the propagation velocity of ultrasonic waves in the crystal. With slightly damped dislocation loops L_N the correction term to the modulus defect is proportional to ω/ω_d. In case the dislocations are highly overdamped, the attenuation rate to the modulus defect ratio naturally shall be the same as for amplitude independent losses [1]. This statement is confirmed by the experimental results to be described in a further paper.

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